THE VIRTUAL CLASSROOM:

AN INNOVATIVE APPROACH TO LEARNING MATHEMATICS

by

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ABSTRACT

New technologies and their pedagogical applications have been of interest to many mathematics teachers and researchers in mathematics education. The increase in popularity and access to the Internet has presented new educational opportunities, as well as new queries and dilemmas. The "virtual classroom" is an innovative approach to using the Internet to support the learning of mathematics. This thesis explores the application of this approach to integrating technology by investigating students' exploration of a particular web site featuring Pascal's Triangle.

The site, Pascal's Triangle, was designed specifically for this study. It attempted to integrate mathematical and historical information with engaging problems and activities, while keeping the mathematical level appropriate for middle school students or preservice elementary school teachers.

Two groups of students participated in the research. The first group consisted of two Grade 9 students who participated in an initial pilot study. The second group involved 38 preservice elementary school teachers enrolled in Mathematics 190 course at Simon Fraser University . The participants navigated through the site, engaged in problem-solving activities presented in the site and provided feedback on their experience.

The results of the study indicated that in using a virtual classroom to structure students' explorations, many disadvantages associated with Web based instruction were eliminated. Students no longer spent hours searching and becoming lost in the vast amounts of information available. However, students still maintained some degree of freedom and could make choices concerning what navigational paths to take.

Another important finding was that students were motivated to learn when using a virtual classroom. Many students commented that this style of learning was "interesting", "fun", and "enjoyable."

The need for more social interaction, both with their teachers and their peers, when using a virtual classroom was a repeating theme in students' reflections.

The results of this research begin to answer some of the questions associated with the use of a virtual classroom. The conclusions reached provide a foundation for future investigations into this innovative approach to learning mathematics.

DEDICATION

To my family

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CHAPTER 1

INTRODUCTION

New technologies are often the impetus for innovation. Perhaps the greatest source of innovations during the past five years has been the computer, particularly when combined with the use of the Internet. Although these technologies chart the direction for change, it is the educators as innovators within the schools that need to have the strength of vision to see how these technologies can be implemented (Houston, 1998).

"Getting connected", "surfing the net", "caught in the web", and "online" are phrases that have acquired new meanings with the introduction of computer technology's latest innovations, the Internet and World Wide Web. The Internet was originally developed in the 1960s for use by the U.S. Department of Defence. This technology expanded to include networks between universities in 1969 and today it is the "information superhighway" linking sites around the world. The Internet consists of a number of features including electronic mail, file transfer abilities, and the World Wide Web. The World Wide Web is "easily the most glamorous" (Starr, 1997, p. 7) of the services provided by the Internet. The potential for this new medium has businesses, governments, athletes, educators and private citizens around the world racing to realise the new opportunities available. However, the introduction of this new technology is not enough in its own right to bring about innovation within the schools. "Society is designed to maintain existing conditions; the tensile strength of change must overcome the tendency to return to a previous status quo" (Houston, 1998, p. 208). In order to adopt this most recent innovation into the classroom, research is needed to provide an insight into how this technology can be used to support teaching and learning.

Research using a “virtual classroom” is not only innovative because it focuses on a new technology, but also because it uses this technology in an innovative fashion. In the few instances where the World Wide Web has been used in education it has most often been employed as a “virtual library” where students go to conduct research. Another alternative application has been the use of the World Wide Web as a “virtual office”, where students go to find information concerning deadlines and requirements for particular courses. Very few sites have been developed as a “virtual classroom”, where students go to learn new theories and concepts. The idea of using the World Wide Web as a “virtual classroom” is in itself innovative and takes educators in a new direction for implementing World Wide Web based instruction. This new style of application allows students to visit a site which was designed specifically to support their learning.

Purpose of the Study

For the purposes of this research, a site that introduces students to Pascal’s triangle and shows students a variety of relationships between Pascal’s triangle and other areas of mathematics was developed.

The objectives for doing this research have developed and grown along with the different stages of the research. Initially, I was most interested with specific design issues of the Pascal’s Triangle web site. Issues dealing with the appropriate amount of text to place on each page, the number of links that should be included, the presentation of the text, and the organisation of the pages within the site were all considered. Research has shown that design factors do affect students' learning and their ability to retain information (Marcus, 1992).

Once the site was designed, a second objective emerged: to examine students' reactions to the site and to analyse the type of learning that takes place when using a "virtual classroom." Students' reactions and reflections were important in examining the variety and types of topics of interest, the depth and motivational aspects of learning, and the changing social interaction between teachers and peers .

The final objective for this research was to explore the impact of site design as shown by students' reactions and reflections. Different pages within the site evoked different responses and an analysis of the pages was important in deciding which design attributes are important in promoting learning. The type and amount of content placed on each page, the number of links and their location within the page, the number of graphics that should appear, and the amount of interactivity within the page were considered. By exploring these relationships, more general recommendations for the design of "virtual classrooms" can be made.

Thesis Organisation

This thesis has been organised according to common themes. Some themes, such as considering how students navigate through the "virtual classroom", were predetermined before beginning the research while others, such as the motivational aspects of using the World Wide Web, emerged as the research progressed.

Chapter One introduces the research and its innovative nature. Research objectives are examined along with the thesis organisation and expectations.

Chapter Two looks at the research that has been conducted by others regarding the use of new technologies within classrooms, in particular the use of the computer and the World Wide Web. This chapter considers the difficulties of adopting new technologies into classrooms, the pedagogies that develop with the introduction of new technologies, the advantages and disadvantages of using the Internet and the World Wide Web, and the variety of applications for the Internet and the World Wide Web.

Chapter Three discusses the methodology used to conduct this research. It looks at the type of methodology that is appropriate when designing and implementing new products and the four stages of product development. The type of data collected from students in the pilot study and how this data aided in making revisions to the site is outlined. Lastly, an overview of the participants in the research group is given along with the ways in which the data collected from this group was analysed.

Chapter Four contains the results and analysis of the data. This chapter is broken into five themes: navigating the Pascal's Triangle web site, motivating students and the World Wide Web, social interaction and the World Wide Web, learning mathematics and the World Wide Web, and evaluating the content of web pages.

The first theme, navigating the Pascal's Triangle web site, was one that was predetermined before the research began. Proponents for learning via the World Wide Web suggest that an advantage of this type of learning is that it gives students a choice in the paths that they take. However, since the Pascal's Triangle web site was structured to direct students within a predetermined framework, it was important to observe the amount of freedom that students exercised while using the site. This section, therefore, focuses on the paths that students took while using the site and their reflections about how the freedom to control their own path affected their learning.

The second theme, how learning through the World Wide Web affects students' motivation, emerged and became apparent as the research progressed. In this section, the increased motivation that students found while using the site is examined along with the factors responsible for the increase.

Another theme that emerged while conducting the research concerned social interaction and the World Wide Web. This section discusses students' need for social interaction with both their teachers and their peers and looks at some possible solutions for how social interaction can be achieved while using "virtual classrooms."

Whether students can learn mathematics through a "virtual classroom" is another theme that was important from the outset of the research. This is the fourth theme that is discussed. Students' solutions to mathematical problems are used to illustrate the degree of comprehension that is possible when learning through a "virtual classroom."

The last theme looks at how the content of individual web pages affects students' learning. Because reactions to each of the pages within the site varied, this section analyses each page in an attempt to determine what content is needed for effective web page design.

Chapter Five, the last chapter of the thesis, is a discussion of the design and use of "virtual classrooms." This discussion uses the results of this research to make recommendations for improving the design of the site along with general recommendations for the design and use of other "virtual classrooms." It also deliberates on whether this research fulfilled its objectives, the questions that emerged as a result of doing the research along with some possible answers to these questions, and the areas of research that still need to be investigated.

Caveats for Innovative Research

As with any innovation, the results of this study only begin to point educators in a possible direction for using the World Wide Web for learning mathematics. As many questions will be raised from conducting this research as will be answered. In doing research using innovations, the purpose is not to arrive at indisputable answers, but to begin to ask questions and to find common themes that may appear. Many issues need to be addressed, not only those relating to the introduction of a new technology such as the Internet and the World Wide Web into the classroom, but also those relating to how best to apply this technology within the classroom. The amount of guidance available from other sources is minimal, as few studies have been done on the use of the World Wide Web within classrooms, especially when considering the unique application of sites such as Pascal's Triangle. "Innovating is a fragile enterprise, affected by changing personnel and shifting community priorities. There are no magic elixirs, road maps, magic wands, or recipes for effective innovations " (Houston, 1998, p. 208).

CHAPTER 2

THE EFFECT OF INNOVATIVE TECHNOLOGIES ON EDUCATION

> Lets work together to meet these goals: Every 8-year old must be able to read; every 12-year old must be able to log onto the Internet; every 18-year old must be able to go to college; and every adult American must be able to keep on learning....
>
> Tenth, we must bring the power of the Information Age into all our schools. Last year I challenged America to connect every classroom and library to the Internet by the year 2000, so that, for the first time in history, a child in the most isolated rural town, the most comfortable suburb, the poorest inner-city school will have the same access to the same universe of knowledge. I ask your support to complete this historic mission.
>
> - Bill Clinton,
>
> State of the Union Address, 1997

This 1997 State of the Union address by Bill Clinton reflects what many educators are feeling about the potential of the Internet and the World Wide Web to transform the art of teaching and learning. Yet, similar refrains have been heard before. As far back as 1922, Thomas Edison expressed similar opinions about the potential of the motion picture when he stated, "the motion picture is destined to revolutionise our educational system" (as cited in Cuban, 1986, p. 9). High expectations were also held for the introduction of the radio in the 1920s; slide projectors, filmstrips, records and tapes in the 1950s; television in the 1960s; and calculators in the 1970s. Although these technologies do have a role to play in today's schools, most people do not consider that their introduction caused a revolution in teaching and learning. Of all the new technologies, the overhead projector has been the most widely adopted for classroom use (Zhao, 1998).

This literature review looks at the technologies that educators have attempted to introduce into classrooms along with some possible reasons for their failures. Many educators (Kaufmann, 1998; Owston, 1997; Zhao, 1998) suggest that the technology itself may not be what is important in determining whether it is adopted, but rather the design and content of the applications that are important.

This literature review will also look at pedagogical change and what changes are likely to take place when the Internet and World Wide Web are introduced into classrooms. Are significant changes necessary in teaching and learning when new technologies are adopted into classrooms?

The advantages and disadvantages of introducing the Internet and World Wide Web into the classroom are a third element that is examined in the literature review. Since this technology is still in its early stages of development, the number of studies available to determine advantages and disadvantages are limited. As well, most studies focus on using the World Wide Web as a "virtual library" rather than a "virtual classroom." If content and design are important to the adoption of a technology, then comparing the use of the Web as a "virtual library" with its use as a "virtual classroom" may not be appropriate.

Therefore, the last element examined in this literature review is the variety of applications available for introducing the World Wide Web into classrooms. One study describing the application of the World Wide Web as a "virtual library" is discussed along with a more innovative application of developing a flexible system for application. Many higher learning institutes use the Web also as a "virtual office" and this application is also examined. Sites that use the World Wide Web as a "virtual classroom" are limited, but a few do exist and some features of these are discussed.

The Adoption of New Technologies into Classrooms

Why is it that many new technologies do not live up to the reputation that precedes them? One reason for the failure of new technologies may be the relationship between the content and the application.

> It is *what* we design and deliver, responsive to the individual requirements and characteristics of each learner, that delivers mastery, competence, and self-sufficiency. The content and its design are important, not the *means* by which we get the material to the learner. It has always been, to rational folk, the focus on the content that is primary (Kaufmann, 1998, p. 64).

These sentiments are echoed in an article by Zhao (1998) who suggests that one of the goals that designers need to focus on is designing content that promotes the adoption of that product.

A number of factors are involved in meeting the goal of educational adoption. One of these is the lack of understanding and appreciation of the power of technology. Workshops, courses, and technology-rich experiences are often used to encourage teachers to use technology. According to Zhao (1998) this is not the right approach. He suggests that teachers should instead be provided with educational applications that bear "explicit connections to their needs and are easy to use" (p. 309).

A second factor that impacts the adoption of technology is the ability of teachers to feel competent using the technology (Zhao, 1998). Technology competence is created not only from increased training but also as a result of simplifying the technology. Therefore, applications should be sought that teachers can use without intensive training.

A third factor in determining whether a technology will be adopted is the ease of access for teachers and students. It is difficult for computer technology in North America to flourish when "the shoemaker's children have no shoes!" (Heterick, as cited in Burg and Thomas, 1998, p. 22). The educational system has not kept up with technology in supplying the computers that students need. Today, most secondary level students have access to computers, however, the demand is still often higher than the supply and few computers are connected to the Internet.

Pedagogical Change and New Technologies

The use of new technologies often changes the way in which we teach and learn (Greening, 1998). The Internet and the World Wide Web are put together in a very non-linear fashion and users do not tend to follow the same linear pathways that they would when using more traditional modes of delivery. A number of changes in education have been brought about by the use of technologies such as the World Wide Web .

-Databases have removed the image of the teacher as the font of all wisdom.

-Search engines place the user in control, permitting them to engage material in natural, interdisciplinary ways, which has implications for

> traditional concepts of compartmentalised discipline as embodied in the roles of the history teacher, the mathematics teacher, etc.
>
> -There is an emerging emphasis on "just in time" learning , rather than a long preapproved list of things to be known (Greening, 1998, p. 24).

With these changes in education, a new pedagogy is emerging. Constructivism, in its belief that children construct their knowledge by assimilating new ideas to preexisting ones, is supported by many of the changes brought about by the computer. However, this does not mean that introducing new technology will necessarily lead to positive pedagogical change.

> Teachers need to be wary of the power of such exciting technology to seduce, and the ease at which emerging theories of education (such as constructivism) may be used to justify the seduction process. To suggest that throwing students at the mercy of the Internet is sound pedagogy on the basis that it adheres to constructivist principles of ill-structured learning environments is to misrepresent the constructivist position (Greening, 1998, p. 30).

If the use of new technologies changes teachers' pedagogical beliefs, will teachers that have teaching styles reflecting constructivist beliefs be the only ones to adopt the computer and the Internet into their classrooms? Studies (Zhao, 1998) have shown that this is the case. If technology use demands too much change in current practices, teachers will avoid using it because of their resistance to change. In the past, implementors of new technologies have tried to overcome this in two ways: one is to educate teachers in an effort to convince them to adopt the new pedagogy and the other is to design "teacher-proof" learning environments that will achieve learning goals set in the classroom. Zhao

(1998) argues that both methods lead to failure in adopting the technology and instead, the technology should be flexible enough to allow teachers with a variety of pedagogies to use it. Then, teachers will learn and explore new ways of teaching and "pedagogical changes are not imposed upon or shielded from teachers, rather they are fostered in the process in which teachers make use of the application" (Zhao, 1998, p. 311).

Advantages and Disadvantages of Adopting The Internet and World Wide Web into the Classroom

The Internet and the World Wide Web may be the latest educational panaceas or they may solely represent the most recent "flavour of the month" (Kaufmann, 1998). Educators evaluate the effectiveness of the World Wide Web in enhancing teaching and learning by focusing on specific questions (Owston, 1997).

The first of these is: "Does the World Wide Web make learning more accessible?" (Owston, 1997, p. 7). At the K-12 level there are three situations where the World Wide Web is facilitating increased access. These are in the areas of home schooling, alternative schooling and extension courses (Owston, 1997). For parents of students involved in home schooling the World Wide Web offers access to learning materials and to peers around the world. Alternative schools use the World Wide Web as a central tool in project-based curriculum. The World Wide Web also has potential in the area of extension courses which are becoming more popular for K-12 students.

A second question in evaluating the merits for improved teaching and learning using the World Wide Web is "Can the Web Promote Improved

Learning?" (Owston, 1997, p. 29). This question is more difficult to answer since in more than 50 years of research on instructional media, no consistent effects on learning have been demonstrated by any of the media (Owston, 1997). Researchers argue that this is because the design of the instructional material, not the medium itself, is responsible for improved learning. Regardless, of the difficulty in evaluating the effect of the World Wide Web on learning, Owston suggests there are three distinct advantages offered by the Web.

The first advantage is that the World Wide Web is compatible with the way students prefer to learn.

> Our children have been raised in a world of instant access to knowledge, a world where vivid images embody and supplement information formerly presented solely through text. They are used to an environment where they control information flow and access, whether through a video game controller, remote control, mouse, or touch-tone phone (Strommen & Lincoln, 1995, p. 1).

A second advantage for improving learning that the World Wide Web has over more traditional media is that it provides for flexible learning (Owston, 1997). Flexibility is evident in offering students the choice as to when, where and how they learn. Distance education courses allow students the choice of learning from home or school in an environment that is self-paced, expert-directed, and time and place independent. Teachers who have used computers in the classroom have also stated that they change how students learn, shifting from a teacher directed environment to a student directed one (Quinlan, 1997). One feature of the Web that aids in the shift from teacher to student directed learning is that of hypertext. "Hypertext on the Web provides the simplest form of user interaction, enabling user control of information" (Starr, 1997, p. 8).

The third advantage the World Wide Web offers in promoting improved learning is that it enables a new kind of learning, placing more emphasis on skills such as critical thinking, problem solving, written communication, and the ability to work collaboratively. The World Wide Web is not the only medium for learning these skills, but many features of the Web make it a natural match for their development (Owston, 1997). "What the Web can offer that traditional media cannot is information that is instantly available, often very up-to-date, worldwide in scope, and presented in a more motivating format for students to explore" (Owston, 1997, p. 31).

The abundance of information presented on the Web makes it useful in developing critical thinking and problem solving skills. Since almost anyone is able to post a page on the World Wide Web, the quality and value of pages vary greatly. This affords teachers the opportunity to have students evaluate the information offered by: weighing evidence, judging the authenticity of data, comparing viewpoints on issues, and analysing diverse sources (Owston, 1997).

Written communication is enhanced when learning via the Web because it gives students the opportunity to post written work to an "authentic audience", whether it is a "key pal" in another country or taking part in a global discussion as offered by KIDLINK (http://www.kidlink.org). When writing to an authentic audience students will be more conscious of their vocabulary, syntax and grammar (Owston, 1997).

In enhancing collaborative skills, Owston (1997) looks at the advantages that the World Wide Web offers for global collaboration. Structuring group projects where students in different geographic locations have a common goal to reach or problem to solve, provides for communication worldwide. One of the largest of these projects is found at KIDLINK where over 48,000 children in 77 countries have participated in global dialogs and projects.

While Owston (1997) points out advantages of the World Wide Web in providing greater access and promoting improved learning, others (Harrison, 1997) believe strongly that these advantages are not enough to justify the adoption of this new technology. Harrison (1997) responds to Clinton's 1997 State of the Union Address by calling his statement a "chicken-in-every-pot rhetoric" and suggests that "overcrowding, under-maintained schools, teacher shortages, violence, bilingual education, basic skills and many others" (p. 32) should be the focus of education.

Some limitations of using the Internet and World Wide Web are discussed by Harrison (1997) and others (Soloway & Wallace, 1997) in their articles on using these new technologies in schools. The following describes a typical problem associated with computer use in many schools.

> Computers have been primarily acquired through one-time monies such as end-year surpluses, grants, and non-continuing budgetary expenditures. Most computers end up in labs, which are not part of an on-going class. Students only have independent access to the machines 45 minutes before and after school. The machines age rapidly and become of little interest to the students visually because their home systems are far superior. Technical staff in the labs is minimal, so the district buys outrageously priced service agreements with overpriced machines. The computers have sat in boxes for up to a year without being set up due to technical difficulties. The labs are cramped just like our normal classrooms

which can cause discipline challenges (Harrison, 1997, p. 33).

Similar problems to this were found by researchers when they studied groups of students completing projects using the World Wide Web (Soloway & Wallace, 1997). However, other disadvantages found in this study were more directly tied to the use of the computer as a "virtual library" and are discussed in the following section.

Different Applications for the World Wide Web

The Web as a "Virtual Library"

Advantages and disadvantages of using the World Wide Web will differ depending on the application used to introduce it into the classroom. The most common application in education is the use of the World Wide Web as a "virtual library". Soloway and Wallace (1997) document some of the problems they observed in their study of over 1,000 students in their early teens, along with 30 teachers and media specialists, in five Ann Arbor, Michigan, middle and high schools.

The first problem was coined "the World Wide Wait" (Soloway & Wallace, 1997). Students in classes that used the World Wide Web early in the day were able to access information relatively quickly and efficiently with waits of 30 seconds or less. But as the day progressed and Internet traffic became heavier, the wait time became longer and students waited 30 seconds to 3 minutes for pages to download. This created discipline problems with students who were expected to sit patiently in front of the monitor waiting for pages to appear.

A second problem observed was the difficulty students had in navigating the Web (Soloway & Wallace, 1997). Students quickly learned how to use features of the browser to "surf" the Web but learning to productively navigate the Web was more difficult and frustrating, even for a generation of students who are used to non-linear video game formats. While navigating, students would become lost because of the overwhelming number of sites they had visited and the amount of information contained within these sites. Different sites which contained links to the same pages would cause students to travel in circles. One student who never realised she was travelling in circles couldn't understand why everyone was saying the same thing.

A third problem (Soloway & Wallace, 1997) was associated with finding information on the Web. Searches for keywords such as "tornado" resulted in information about a major league baseball pitcher nicknamed "Tornado Boy" and for keywords such as "volcanoes, water life" resulted in 14,000 matches. When using the World Wide Web to find information, teachers are faced with the added task of spending time teaching students how to search.

The problem of censoring the Web and monitoring inappropriate material for students is a fourth limitation (Soloway & Wallace, 1997). One student in the study (Soloway & Wallace, 1997) performed a search on "rocks" and was presented with a site called "SexRocks" as the fourth hit in a list of thousands. Another student, searching for information about weather, typed in "storm front" and came up with a list that included the site for White Nationalist Web Resources published by White Supremists.

The last problem discussed by Soloway and Wallace (1997) concerned students' perceptions about how to find answers to problems. The study found that Web resources feed the misconception that answers are readily available and every question has a simple answer. This leads to educators questioning if we are really trying to teach students that the way to solve a problem is to ask someone else.

The Web as a "Systems" Approach

Zhao (1998) gives an alternative suggestion for using the Web in classrooms in which he designs a "system" that can be used by both teachers and students. His rationale is that:

> the process of implementing an innovation is, in essence, a process of re-creation in which teachers and students reinterpret the innovation in their own terms. The realisation of any innovation often reflects a set of compromises between old and new ways of doing things. In other words, to promote adoption, an innovation must allow its users to re-create it so as to meet their needs (Zhao, 1998, p. 315).

The system that Zhao (1998) designed is called eWeb and it consists of six components. These components are intended to give teachers the flexibility to adapt the eWeb system to meet their particular needs. The first component is a Forum where students communicate in an environment where messages are catorgorised and stored in databases for future reference. The second component is a Bulletin Board intended for brief messages that do not need in depth or ongoing discussions. The third component is the Chat room where teachers and students participate in synchronous communication or real-time chats. The fourth component of eWeb supports teachers in allowing them to create and administer tests and exercises. This component supports multiple-choice, fill-in-the-blank, true-or-false, word and sentence scramble, and long answer questions. A fifth component also supports teachers allowing them to develop and manage instructional materials. Teachers can copy and paste any text document into the main document window and then add links to sound, images and video clips. The last component of eWeb is the Homepage Maker where teachers use an interactive form to create a homepage for a specific course and students use different forms to create their own homepages with personal and course related components.

The use of the eWeb site has been undergoing research since October, 1996 (Zhao, 1998). As of May, 1997, 286 individuals had applied to become administrators of an eWeb site. Registered administrators can create new groups with unlimited numbers of users so an exact number of users is not known. Feedback from users has shown that the Bulletin Board is the most frequently used component of the site. As well, server logs show that users tend to focus on using one component of the site. For example, one teacher will focus on the test and exercise builder while another teaching languages may use the Chat room to practise foreign language skills.

Feedback from users about difficulties occurring in operating a system such as eWeb has been minimal (Zhao, 1998). Only 17 users have asked for help in understanding features of the system. However, users have reported software and hardware problems and problems with busy signals coming from the server.

One aspect of the eWeb system that users suggested needs attention is the amount of hypertext links found on the screen (Zhao, 1998). Users suggested that too many hypertext links on the screen become confusing. An example of this is found in the Forum component of eWeb where a user reading a

message can choose to read, comment, or evaluate the message; list all messages or list his or her messages only; search for specific messages; delete messages; list all those who have read a particular message; sort all messages in four different ways; or link to the various other sections of eWeb (Zhao, 1998).
In summarising the impact eWeb has on education Zhao (1998) states :

> The development of eWeb represents one of many attempts to translate the technological possibilities of the Internet, particularly the Web, into more effective learning and teaching. As one of the pioneering Web-based integrated and interactive education environments that place an emphasis on adoption, eWeb has some implications for future research and development of educational applications. First, it makes use of the current server-client technology, which simplifies the technology for end users and thus helps promote adoption. Finally, preliminary data suggest that eWeb's explicit intention to support education seems to help teaches start using the Web because eWeb allows them to see not only the potential but also the immediate benefits (p. 324).

<u>The Web as a "Virtual Office"</u>

Many schools, especially universities and colleges, are setting up Web pages to provide students with information about the courses that they offer. This use of the Internet is best described as a "virtual office". Fetterman (1998) outlines the many uses of the Internet and the World Wide Web and gives a description of how he has used the Internet to set up his "virtual office." He uses folders on specific topics to organise the information so that the folders can be accessed by colleagues and students from around the world. Other instructors who use the Internet and World Wide Web as a "virtual office" use it in a slightly different manner. Students visit the Web to find out course requirements, lists of assignments and their due dates, class notes and activities, and postings of their grades. Some "virtual offices" even allow students to hand in assignments through the Internet eliminating the need to travel to the college or university.

The Web as a "Virtual Classroom"

The concept of using the Internet and the World Wide Web as a "virtual classroom" is still in the early stages of development. Few projects exist that have used this type of application. One of the few projects has been sponsored by the British Columbia government which has granted funding to a group of individuals involved in developing a web site called the Brainium. The Brainium is specifically tailored to the British Columbia Junior Science curriculum. When visiting the site, students can choose from a variety of topics within the Junior Science curriculum to explore via the format of a game that features graphics, sound and animation. As well, the site supports links for teachers that provide relevant resources and teaching materials. One limitation to the Brainium site is the ability to access it with older technology. Because of the large amount of memory required for the graphics, sound, and video clips, downloading is slow and even impossible for those using older technology.

Conclusions

A number of factors need to be considered when studying the introduction of new technology into the classroom. First, the technology must be adopted by educators. This process has many barriers and applications must be designed that can meet the needs of teachers and learners. Second, pedagogical change will likely occur with the introduction of a new technology and this issue needs to be addressed. Third, the advantages and disadvantages of the technology need to be understood in order for educators to decide how it can best meet their needs. These advantages and disadvantages will likely vary with the type of application that is used. Therefore, educators also need to examine the variety of applications that are possible.

> No doubt further research and development on the application of the Web to teaching and learning is needed. Nonetheless, in the meantime, it merits serious consideration as we search for ways to revitalise and enhance what we do in our schools (Owston, 1997, p. 33).

CHAPTER 3

METHODOLOGY

The methodology I used for this research follows the four stage development process for designing new products suggested by Romberg (1992, p. 58).

> It is common, especially in mathematics education, that individuals or groups create new products intended to improve teaching and learning. The products may be new instructional materials, instructional techniques, or instructional programs. The development of a product involves an engineering process of inventing parts and putting them together to form something new. There are four stages to the development process: product design, product creation, product implementation, and product use.

The first three stages of product design, product creation, and product implementation were the focus for this research with the fourth stage of product use reserved for the future. This process, however, was cyclic rather than linear since product implementation provided feedback that was used to modify the product design.

In discussing the methodology I focus on a number of different topics. In the first topic I look at the initial design of the Pascal's Triangle web site and the motivation behind its design. Problems that exist with most current web sites are discussed and I introduce a site that gave inspiration and acted as a model for this project. In the second topic I look at the pilot study that took place in the fall of 1997: the purpose, the participants, and the results which were used to modify the Pascal's Triangle web site are discussed. In the third topic I focus on revisions which were made to the Pascal's Triangle web site. I look at the specific revisions which took place and the rationale behind these revisions. In the fourth topic I focus on the participants involved in the main body of research in the winter of 1998. The mathematical and computer backgrounds of the participants and how they became involved in the study are discussed. In the fifth topic I look at the three different ways in which data collection occurred: the double-entry journal, the mathematical problem, and the student reflections. In the last topic I focus on how these three different types of data are used in analysis. I conclude the methodology chapter with a brief discussion of the cyclic nature of the methodology involved in designing new products.

Initial Site Design

The development of the web site used in this study began in the summer of 1997. My experience with the Internet and the World Wide Web had shown that most sites on the web were designed for use as a "virtual library." Existing sites were mainly structured as sources of information that would be most useful for students involved in a research project. However, this use of the Internet has the disadvantage of requiring a large investment of time searching to find information that is relevant and at an appropriate level for student use. This time investment is made worse when coupled with the problems of slower or malfunctioning computers. My experiences with using the Internet as a "virtual library" had often been frustrating and I was hesitant to use this type of application with a group of students.

Inspiration for the Pascal's Triangle web site came from an existing web site that was designed more as a "virtual classroom" rather than a "virtual library." This site (available at: http://forum.swarthmore.edu/alejandre/magic.

square.html) revolves around the topic of Magic Squares and was written by Suzanne Alejandre, a middle school teacher from southern California. The site is unique since it presents information in a format that allows the user to navigate through a series of pages which include both instructional information and interactive activities. Here, students can learn a topic in mathematics similar to how they learn in a classroom. However, the variety of activities and the depth in which the student can explore the topic seems to be much greater than they would get from using a textbook in most classrooms. This site gave me the inspiration to expand my view of using the Internet and World Wide Web solely as a "virtual library" to include the more innovative idea of using it as a "virtual classroom."

The Magic Squares site uses a thematic approach and I decided to structure my site around a theme as well, since this would allow me to tie together many aspects of mathematics and take better advantage of the interactive nature of the Internet. I chose the theme of Pascal's triangle because I was familiar with many of the relationships connecting the triangle with other areas of mathematics. I believe that the historical aspects of mathematics are important in teaching and the theme of Pascal's triangle also gave me the opportunity to link with many historical topics.

Once I had decided on a theme for this site, I began to consider design options. Many of the decisions that I made about the design of the site were based on research conducted by Marcus (1992). Some of the issues addressed are those regarding colour, typography, layout, and graphics.

The initial web site that I designed consisted of six pages. The first page was simply a table of contents that contained interactive links to each of the remaining five pages. The titles of the remaining five pages were: Blaise Pascal, Pascal's triangle, Chinese triangle, Pascal and Probability, and Pascal and the Fibonacci sequence. The structure for each of these pages remained consistent, with each page starting with a title and image of Pascal's triangle at the top of the page followed by a two column table below that organised the rest of the information. I tried to keep the amount of information on each page minimal so that students would not have to scroll through volumes of text and to eliminate long waits while pages were loaded. Links to external sites were provided allowing students to obtain more detailed information on topics they were interested in pursuing. At the bottom of each page were links that would go to the previous page, the next page, or back to the contents page.

The Pilot Study

The initial site was piloted with two students in December, 1997. I felt that before I spent more time developing the site I needed some feedback from potential users. I chose to work with just two students since this allowed me to make detailed observations on the use of the site and to get feedback through an interview format.

Both students that piloted the site were Grade Nine mathematics students. Jack was enrolled in modified Math 9 and had weaknesses in some of his basic math skills. His computer experience was moderate to advanced and he had an interest in the Internet and the World Wide Web. He had been involved in designing his own web page and had a basic understanding of HTML programming. The second student, Zachary, was enrolled in regular Math 9 but was working independently on some Math 10 curricula. He had some experience with computers but was not as experienced with the Internet and the World Wide Web as was Jack .

I met with both students individually and worked with each of them for approximately two hours. The students were given the site address and asked to explore the site for approximately one hour. During this one hour period I observed the students while they worked and was available to answer any questions they might have. The second hour with each student was spent interviewing them about their use of the site.

In the interviews with Jack and Zachary, we discussed issues such as the page setup, the type and quality of links, the type of activities on the pages, the amount of "wait" time when using the site, and the type of approach teachers should use when presenting the site to a class. In addition, the students gave general comments concerning their likes and dislikes of the site, what they had learned from the site, and what topics they found most interesting.

The responses of the two students were fairly similar regarding their views of the site. The main difference was in the type of learning that occurred. Jack visited every page in the site and read over the information provided quite quickly, whereas Zachary spent more time focussing on the content of the pages and did not make it to all pages within the site. Because of this, Jack remembered only interesting facts and antedotes while Zachary had a greater depth of learning about some of the concepts presented.

Revisions to the Site Design

The feedback I obtained from Jack and Zachary was extremely useful and allowed me to work on redesigning my site. The interviews highlighted two areas where major changes needed to take place. The first concerned the navigation of the pages that the students followed when using the site. It was clear from the pilot study that in order for students to grasp the connections between Pascal's triangle and other areas of mathematics, they needed to start with at least some basic understanding of Pascal's triangle and how it was formed. In the case of Jack and Zachary this did not occur. Both students started with pages that were more advanced and required some background knowledge in order to be fully understood. Therefore, the first goal in redesigning the site was to restructure it so that students would follow a path that led them through the basic information before embarking on alternate pathways. To accomplish this goal, I changed the index or starting page to one that explained what Pascal's triangle was and how it was formed, rather than having students start with a contents page. From this page students were directed to a second page which included an activity using modular arithmetic to colour Pascal's triangle. This second page was not crucial in student's basic understanding but I chose to lead students here since both Jack and Zachary had enjoyed this activity.

The second goal in redesigning the site was to encourage participation in the problems and activities that were presented. Upon observing Jack and Zachary, I realised that if a problem or activity required students to move away from the computer to use a pen and paper, then the students would skip over the problem or activity and move on to the next page. I also noticed that both Jack and Zachary were attracted to the interactive features of the site such as the page which allowed interactive colouring of the triangle. Because of this, I decided to attempt to increase the participation in the problems and activities by including as much interactivity within the site as possible. Introducing interactivity proved technically difficult in many cases and it still remains one of my stumbling blocks since interactivity for many purposes requires knowledge of new technologies such as Java programming. I was able to increase the interactivity of the site in some areas by using forms that allowed students to type in answers to problems and by linking to some sites where there is interactivity. These interactive sites that related to Pascal's triangle included: the Colouring Pascal's Triangle site, the Tower of Hanoi site, and the Chinese Abacus site.

The new Pascal's Triangle web site that emerged after the pilot study consisted of 13 pages with links to another 10 external sites. Appendix A contains copies of each of the pages from the site. The site begins with a page that gives a brief history of Blaise Pascal and then leads students through the process of developing the triangle. There is an interactive form that allows students to input numbers into a partially blank Pascal's triangle and a link to a second page that allows students to check the numbers that they entered.

The third page in the site describes how the concept of modulus can be used to colour the triangle resulting in a variety of patterns. This page is linked to an external site where students can use the computer to colour the triangle by inputting the modulus, the number of rows, and the size of the final image. If students choose to use modulus three to colour Pascal's triangle, the numbers in the triangle that are evenly divisible by three are coloured black, the numbers that leave a remainder of one are coloured red, and the numbers that leave a remainder of two are coloured green. Also found on the third page is a quiz asking students to identify what modulus has been used to colour six different triangles. The quiz includes an interactive form that allows students to input their answers with a link to a fourth page for students to check their answers.

After the Colouring Pascal's Triangle page students are led to a contents page which consists of four options to follow; a page on Blaise Pascal, a page on the Chinese Triangle, a page on Pascal and the Fibonacci Sequence, and a page on Pascal and Probability. Each of these four pages contains links to external sites that I chose because their content was effective and appropriate for junior high school students. I have also tried to include activities on each of the four pages such as the Fibonacci jigsaw puzzle and the game based on the Tower of Hanoi puzzle.

Appendix B shows a site map of the complete Pascal's Triangle web site in its present form. Pages that are part of the site are shown using regular shaped triangles and pages that are directly linked to the site are shown using rounded rectangles.

Study Participants

The pilot study was followed by the main body of research that involved 38 MATH 190 students from Simon Fraser University. The MATH 190 course is a core course for those planning to pursue a teaching career at the elementary school level. Prerequisites for the course include a grade of at least a "C" in Mathematics 11. Although the course is offered through the department of mathematics, it can not be used toward degree credits for a Mathematics minor, major, or honours degree.

The students in the MATH 190 course were given the choice of doing two out of three projects as part of the requirements for completing the course. The three projects involved: doing a mathematical poster, solving a mathematical problem, or using the Pascal's Triangle web site. Of the 70 students that were enrolled in the class, 38 students chose to explore the Pascal's Triangle web site as one of their two projects.

Because of the number of students involved, I chose not to do in-depth interviews with each one. Students provided written feedback as part of the requirements for the project. This feedback was in the form of a double-entry journal, a solution to a mathematical problem, and a reflective summary.

Prior to starting the project students were asked to give an indication of their experience with the Internet. Not all of the 38 students who participated in the project were represented since the survey was conducted during a MATH 190 class prior to the start of the project. However, the results showed that of a total of 27 students, approximately one-third of the students had never used the Internet, one-third of the students had used the Internet several times, and one-third of the students felt comfortable using the Internet.

Tutorial sessions were offered to students, but only 11 students took advantage of them. None of the students who came to tutorials seemed to have problems with the actual operation of the Netscape browser as they navigated the site, however, a couple of students needed a quick five-minute introduction. For most of the students who came to tutorials, the mathematics contained within the site seemed to be unfamiliar. Students needed help with some of the mathematics, particularly with the concept of modulus and how to apply this concept to Pascal's triangle.

Data Collection

Each student who chose to participate in exploring the Pascal's Triangle web site was given a project outline that discussed the three requirements for completing the project. A copy of the instructions that they were given can be found in Appendix C.

The first requirement was for the students to keep a journal of their activities as they used the web site. Students were given a sample of a double-entry journal which showed a format where the address and title of each page were recorded on one half of the paper and comments about the page were written on the other half of the page.

The second requirement for the project was to complete one of three mathematical problems that were presented on the site. The first mathematical problem dealt with modulus and how it can be used to colour and find patterns in Pascal's triangle. The second problem was to explain how Pascal's triangle can be used to solve the Tower of Hanoi problem. The third problem involved the Fibonacci jigsaw puzzle where a rectangle is rearranged to form a square whose area is less than the starting rectangle.

The last requirement to complete the Internet project was for each student to write a one to two page reflection of their experiences while using the site. Students were given a list of questions that they could answer and they were encouraged to add their own thoughts and queries.

Of the 38 students that participated in the Pascal's triangle project, only 35 handed in completed assignments. Two of the students did not go to the web site that was created for them to explore but instead did a search on Pascal's triangle and explored sites that they found as a result of the search. A third student completed only the reflections part of the project and did not complete the journal or mathematical problem sections. As a result of this, data from the journals and the mathematical problem will be based on a total of 35 participants and data from the reflections will be based on a total of 36 participants.

Data Analysis

The three different components of the project provided the data for analysis. Key words and phrases used by students were recorded for both the double-entry journals and the reflections. These key words were organised by page for the double-entry journals while the data from the reflections only recorded the total number of each keyword or phrase used. The mathematical word problems were analysed according to the depth and type of learning that was exhibited.

The analysis of the data is organised according to five themes, with each theme using different parts of the data collected. In addition to the data from the three components of the project, insights gained from the informal tutorial sessions were used.

The first theme, which looks at navigating the Pascal's Triangle web site, uses data from students' double-entry journals. Navigational maps were made from each student's responses about the pages that were visited and the order in which they were visited. Three representative navigational maps are provided in Appendix D. There seemed to be some errors in the journals since comments in the journals sometimes indicated that students had visited a page that had not been recorded. As well, the order of pages sometimes seemed to be incorrect since students would speak about activities found on a certain page before they recorded having visited that page. In making the navigational maps I read the comments and if students indicated that they had visited a page not recorded I would make the correction and include this page in the navigational map.

The second theme in the analysis looks at motivation and the World Wide

Web. It uses both the double-entry journals and the reflections in the analysis. Both sources gave evidence of increased motivation; with the reflections providing a more general account of the students' feelings and the double-entry journals providing more specific information that aided in determining why the pages were motivational.

The third theme, which discusses the World Wide Web and its effects on social interaction, also uses data from both the double-entry journals and the reflections. The reflections provided general feedback about the students' desire for social interaction. The double-entry journals mostly gave data to support the need for teacher help, especially on the pages where the students experienced difficulty.

The fourth theme looks at the learning of mathematics via the World Wide Web using data from the mathematical problems . Students' responses to the problems were analysed in order to see the depth of learning that had taken place and how this learning was different from that which would have occurred in the traditional classroom.

The last theme for analysis focuses on the content of the pages within the site in order to determine what makes web pages more or less effective. This theme uses the data that was collected from the double-entry journals and makes comparisons between comments that were made about the main pages found within the site. Informal observations from the tutorial sessions were also valuable for this theme since they provided feedback about what students seemed to enjoy while using the site and what caused them frustrations.

The Cyclic Nature of the Methodology

The last stage of the methodology outlined by Romberg (1992) for the design of new products has not been completed as part of this research. This stage involves actual use of the product. In the case of the Pascal's Triangle web site I would want to use it with junior high school mathematics students. This did not occur for two reasons; the technology was not yet available for widespread use by students in most high schools and I wanted to refine the site before using it on a larger scale.

Even though widespread use of this product was not part of the research, I hope to use the Web site with Grade 8 mathematics students sometime during the 1998/99 school year. The school where I currently teach is undergoing wiring for the widespread use of the Internet, so the problem of access will be solved.

One of the advantages of using the World Wide Web is the ease with which changes can be made to existing pages. Results of the pilot study provided feedback to make changes to the original site and I anticipate further changes to the present site. Because of this, I see the methodology of product design, product creation, product implementation and product use continuing to be a cyclic process with changes being made as more feedback is obtained.

CHAPTER FOUR

RESULTS AND ANALYSIS

The data provided by the research is analysed according to themes. Some of these themes, such as students' navigation through the Pascal's Triangle web site, were predetermined before starting the research. Other themes, such as the motivational aspects of using the Pascal's Triangle web site, emerged as the research was conducted. In all, five themes are developed in this chapter.

The first theme discussed is navigating the Pascal's Triangle web site. This theme focuses on the pathways that students took as they used the site. Issues such as the quantity of both internal and external pages visited and the order students chose in visiting these pages will be examined. I explore the relationship between student choice and motivation and consider whether designing a site such as Pascal's Triangle to structure students' navigation impedes their ability to have choice in their learning.

The second theme emerged from the data contained within the double-entry journals and the reflections. This theme explores the relationship between motivation and Pascal's Triangle web site. Comments from the data are used to show that motivation is increased when students learn via the World Wide Web. In addition, I use comments from the double-entry journals to analyse what aspects of the site are important in providing this increased motivation.

The third theme also emerged from the data contained within the double-entry journals and the reflections. Students' responses showed that social interaction was missed when learning mathematics via Web. Students wanted more social interaction when teacher support was needed to understand the more difficult concepts and when peer interaction was needed to discuss new ideas. Some students gave possible solutions for increasing social interaction when using web sites and these solutions are also explored.

The fourth theme was predetermined from the start of the research. This theme focuses on the style and depth of learning that takes place when students learn mathematics through the World Wide Web. In this analysis, data from the three mathematical problems is used to describe the depth and nature of students' learning. Sample works from students are used to illustrate the learning which took place.

The last theme evaluates the content of individual pages within the site in order to learn how content contributes to the success of each page. Data from the double-entry journals is used for this analysis. The analysis is used to make recommendations for changes to the existing site and general recommendations for the design of "virtual classrooms."

Navigating The Pascal's Triangle Web Site

One of the main advantages that the World Wide Web offers for students engaged in exploring a "virtual classroom" is that it offers choice in the pathways that students take. This ability to choose supports the foundational premise of constructivism (Strommen & Lincoln, 1995). By choosing the paths they take, students can build on their interests and choose pathways for which they have preexisting notions.

However, there is also a disadvantage associated with giving students the ability to choose and control the path of their learning. The Internet has grown exponentially, increasing from 213 host computers in August, 1981 to 2,217,000 hosts in January, 1994 (Starr, 1997). With this exponential growth in hosts, the amount of information available to students has become immense. This creates problems for students involved in Web-base research: first, it is difficult to find the information that they are looking for; second, it is easy to become sidetracked and to get lost in the maze of the Web; and third, students may not have the prerequisite information required since topics are presented in a non-linear fashion. By creating a specific site for students wishing to learn mathematics through the Web, I hoped to avoid the disadvantages of Web-based learning while still maintaining the advantage of giving students choice.

Pascal's Triangle Web Site's Effect on Student Choice in Navigation

In order to evaluate whether students were exercising their ability to chose their own pathways while navigating the Pascal's Triangle web site, I examined variations within the students' navigational maps. These maps were made using information from the double-entry journals in which students were asked to record their travels as they navigated through the site (see Appendix D for three representative maps). Twenty-four, of the thirty-five students, followed the same or similar paths for the first two pages of the site until they came to the contents page (More Pascal's triangle). After these first two pages, the choices students made were quite varied, both in the pages they visited and in the order in which the pages were visited. This variation in navigation is evident in that none of the students progressed through the site in exactly the same way.

Another interesting phenomena that arose from the data provided by the navigational maps, was that students did not spend much time engaged in exploring a particular area of interest. While students would visit pages within the site, they seemed reluctant to use the external links to visit pages outside of the site.

Nine links to external sites were included on different pages of the Pascal's Triangle web site. I did not include a tenth link to an external site in my count because it was part of the Colouring Pascal's Triangle page and students were automatically taken to this site when they entered information into the modulus table (see appendix A; Colouring Pascal's Triangle page). The most popular of the nine external links was one that took students to a site where they could play the interactive Tower of Hanoi game. Twenty-four students visited this link. One reason for the popularity of this site was that it contained information students needed to complete one of the three mathematical problems included in their assignment. If this link is eliminated, then only 17 students visited external links provided within the site. Five students did not visit even one external link.

Students also had the opportunity, when visiting external sites, to continue exploring in their area of interest by linking to further sites provided on these pages. Again, the most popular of these was a link found on the Tower of Hanoi site that took students to pages that gave hints on how to solve the Tower of Hanoi problem. In spite of the usefulness of this site only six students visited it. Overall, the number of students that branched out along a certain area of interest by exploring links provided on the external sites was very small, with only nine students doing so.

There are a number of possible reasons why students seemed reluctant to pursue a particular area of interest when they used the Pascal's Triangle web site. The first of these may have been the approach that was taken in this study. Students were given an assignment which asked them to explore pages within the Pascal's Triangle site and which did not specifically ask them to explore other external links. A second reason could have been a result of the large quantity of pages and information contained within the site. The time allowed for the exploration of external links may have been inadequate. Lastly, it could be that structuring the web by using a "virtual classroom" caused students to become focused on the information provided within the site and to lose the motivation to explore.

Student Choice and Motivation

Evidence that having control over navigation added enjoyment to students' learning was found in many comments, both in the student's double-entry journals and reflections. Students varied in their reactions to different topics within the site, with at least one student using words such as "interesting", "surprising", and "engaging" to describe each of the seven main pages. The most popular page was the Fibonacci Investigations page with nine students describing this page as "interesting", "surprising" and "engaging". These words were used by six students to describe each of the Blaise Pascal and the Pascal and Fibonacci pages, by four students to describe the Pascal and Probability page, by three students to describe the Pascal's Triangle page, by two students to describe the Chinese Triangle page, and by one student to describe the Colouring Pascal's Triangle page.

Some students spoke specifically about the motivation that resulted from being able to have navigational control. Kathleen wrote: "The power to learn is in your own hands which can really motivate people to achieve higher levels of education which could result in a higher percentage of people receiving and finishing an education program." Another student, Kimberly, related the freedom she found in exploring math on the Internet to the nature of mathematics itself.

> The internet site seems to exemplify mathematics itself, such that exploring the web site was like attempting to solve a difficult math problem -- you must let your mind wander and be willing to explore a variety of paths and approaches by which to solve the problem. Finally, the variety of links into a multitude of other mathematical topics and subjects helped reinforce the idea that math is built on many layering concepts, which support and explain one another.

Lastly, Betty, discussed the freedom offered by learning over the Internet and said that "learning over the Internet gives the student the freedom to explore beyond the boundaries of mathematics. Information will be endless that any mathaholic's heart will be satisfied."

Maintaining Navigational Freedom in the "Virtual Classroom"

The goal behind designing Pascal's Triangle web site as a "virtual classroom" was to try to capitalise on the advantages offered by the Internet and the World Wide Web while avoiding some of the pitfalls.

Most advantages are based on the students' ability to have choice in their learning. Data showed that the ability to chose was partially maintained when students used the Pascal's Triangle web site. Students demonstrated that they exercised their ability to choose by the variety of different pathways they took when exploring the site. This was supported by many students who commented that the reason they chose to visit some pages before others was that they found some topics more interesting than others. However, the structure of the site seemed to restrict students in their ability to explore areas of interest outside of the site. This limitation may have been partially caused by the approach that was taken when using the site and research that uses a more open-ended approach may reach different conclusions.

Eliminating Navigational Obstacles with a "Virtual Classroom"

There are many disadvantages to using the Internet and the World Wide Web that this research tried to avoid by using a "virtual classroom." Among these disadvantages are problems with the time required to find information on the World Wide Web, the tendency for students to become sidetracked, and the students' lack of adequate background knowledge.

One would expect that the time required to find information when using a "virtual classroom" would be greatly reduced or even eliminated. This assignment's approach was very different from the "virtual library" approach, where students are given a research topic and asked to conduct a search for information . Since the information and appropriate links were already provided, it was not necessary for students to conduct searches. However, it is interesting to note that three students still did conduct a search. All of these students conducted their searches after having browsed through the site and two of the students described their reasons for doing so. Joanne said that she wanted to find out if "Pascal was really that popular" and Mary said that she wanted to see what "other math pages were like."

The second navigational disadvantage to using the World Wide Web in education is the tendency to become sidetracked. While I did not ask students to record the amount of time spent "lost" in the Web, data from the double-entry journals and the reflections can be used to draw some conclusions. Data collected from the students' navigational maps showed the number of external sites that students visited was surprisingly low. The average number of external sites visited per student was 2.4 and 14% of the students did not visit any external sites at all. The small number of external sites visited is even more significant in that one of the problems that students could choose required visiting an external site. The small number of external sites visited suggests that students are not spending time "lost" in the web when using the "virtual classroom." This is summarised in the following reflections written by Kevin:

> The Internet provided an almost instantaneous access to a broad range of study material. The links from the web site on Pascal's triangle were equally informative and convenient. The relative ease in research readily sped the learning curve as less time was spent wandering around the vast corridors of the SFU library.

The third navigational disadvantage when using the World Wide Web for learning mathematics, was not evident until I conducted a pilot study with two Mathematics 9 students. It became clear, that in order for the students to understand much of what was discussed on the various pages within the site, they required some prerequisite knowledge about Pascal's triangle. Because of this, the site was restructured so that students would first visit a page that discussed Blaise Pascal and how to generate his triangle. Following this page was one that discussed how the concept of modulus could be used to colour and find patterns in Pascal's triangle. It was my goal that students would visit these two pages before reaching the contents page (More Pascal's triangle) where they could choose which pages to visit next. I hoped to achieve this goal by providing conspicuous links that said "next page" and "back to previous page" to restrict navigation. Links to any other pages in the site were small and placed at the bottom of the page in an attempt to make them inconspicuous. This goal was partially achieved since 24 students visited the Pascal's Triangle page and Colouring Pascal's Triangle page before proceeding to the contents page or other pages. Three of the eleven students who did not follow the path along which I tried to direct them did so because they misinterpreted the instructions given to them at the beginning of the assignment. These instructions gave students the address of the starting page for the assignment but they also gave the address for the Colouring Pascal's Triangle page in the sample double-entry journal. These three students, therefore, started at the Colouring Pascal's Triangle page rather than the intended introductory page. If these students are eliminated from the data, then the number of students that visited the introductory pages before going on to the rest of the site becomes 24 out of 32 or 75%. The remaining eight students purposely chose to change the order with which they progressed through the site. Some students gave reasons for this with many being similar to Evelyn's, who said: "after reading the first page on Pascal's triangle and how it was discovered I then read the page titled Blaise Pascal. At this point I was more interested in the development and the person behind the discovery."

In evaluating whether students acquired the prerequisite knowledge needed for the site, it is not enough just to consider whether students visited the introductory pages. It is also important to test whether these students have understood the basic concepts that were introduced on these pages. Because of this, I included a partially completed version of Pascal's triangle at the bottom of the first page that students could complete and then check their answers. This interactive activity seemed successful since only two students indicated that they had problems with this page, and 17 students used words such as "easy", "understandable", and "simple" to describe this page.

Summary: Navigation Through Pascal's Triangle Web Site

The data from this research shows that students did exercise some choice in which pathways to follow when using the Pascal's Triangle web site. There is also evidence to suggest that these students enjoyed having control over their learning. However, in order to find solutions to some of the disadvantages that result from using the World Wide Web as a medium for learning, some choice was eliminated and students explored external sites less than they likely would have if using the web as a "virtual library."

Many of the navigational disadvantages of using the World Wide Web have been eliminated by using the web as a "virtual classroom." The majority of students did not spend much time searching for information or "lost" in the web. It is interesting however, that even though the site gave students information and links to other sites, the curiosity of some students still led them to conduct searches.

The requirement for students to have some background knowledge was only partially solved. Not all students followed the pathway that they were directed to at the beginning of the site. An alternative solution could have been to eliminate links to any other pages on the first two pages, but this option would have restricted students that were visiting these pages for a second and third time.

Motivating Students and the World Wide Web

Motivating students within a mathematics classroom has traditionally been a challenge for mathematics teachers. Mathematics is often taught as a process of memorising rules and algorithms and learning how to apply these rules in a variety of situations. Often, students' expectations from mathematics are that there is a right or wrong answer and that the purpose behind doing mathematics is to arrive at the right answer. Extending mathematics to include the study of historical figures, legends involving numbers, mathematical "magic" and "tricks", and mathematical games, is something that does not often occur in the traditional mathematics classroom.

The fact that students found using the World Wide Web to learn mathematics motivational is a theme that emerged as the research progressed. Comments such as: "this was fun!", "when using the web to explore math, I found myself getting into the problems more than if I was staring at the book", "this program should be used with older children because it is easy and fun", and "all in all this was not only a fun project but an eye opener to many things about learning", gave enthusiastic testament to the enjoyment many students had while navigating through the pages of the site. These comments are indicative of opinions expressed by many students. Twenty-five of thirty-six students used words such as "fun", "exciting", "entertaining", "interesting" and "rewarding" in their reflections to describe their experiences while using the web.

Many students commented that they had fun exploring this site in spite of problems that they encountered during their explorations. Laurie, who was particularly frustrated by the lack of teacher feedback when she encountered problems, wrote: "although this particular project was not successful for myself, I enjoyed the experience and hope that in other circumstances a similar project might arise." Andrea, who had complaints about the slow speed of her computer and the lack of teacher support, wrote : "regardless, I did enjoy surfing the different sites and exploring the various pages." These comments by students about the fun they had while working on the site suggests that there must be a motivational component to learning over the web.

Legends and Their Impact on Motivating Students

The page that was most often described as being "fun", "exciting", and "entertaining" was an external site put together by the University of Toronto where students were asked to solve the Tower of Hanoi problem. This problem involves a legend in which monks move fragile disks from one location to another. The site describes the legend to the students and then allows them to solve the problem using a game format where students interactively move the disks. Students can choose the starting number of disks allowing them to make the game easier or more difficult.

I believe that the Tower of Hanoi site incorporates a number of factors that make this web site motivational. One of these factors is that the Tower of Hanoi game involves a legend. This incorporates the element of story telling into learning mathematics. My own experience with teaching junior mathematics is that students enjoy activities that use legends and story telling to enrich their learning. The element of legend is also incorporated into Suzanne Alejandre's site on Magic Squares after which the Pascal's Triangle site is loosely modelled. In her site, Suzanne tells the legend of Lo-Sho and how he saw a magic square in the etchings on the back of a turtle.

Many students indicated an interest in the historical information and legends presented in the site. Joanne wrote the following about the Tower of Hanoi site: "I chose this math problem because I am really interested in the historical aspect to the problem and how they could have made the Tower." Another student, Kathleen, summarised the historical attributes of the Pascal's Triangle web site when she wrote: "This particular program not only focuses on the present issue in great detail but it also includes very interesting information regarding the background information that normally would be skipped or rarely addressed."

Interactivity and Motivating Students

A second reason contributing to the popularity of the Tower of Hanoi site was its interactive nature. On this site students are involved in an interactive manner beyond using links to turn pages. This use of interactivity is supported by Cates (1992):

> A hypermedia product is not an electronic book. It works to involve learners, to challenge them, and to help learners to make decisions and for their decisions to be meaningful. Within each subject area, developers need to assure that the interactions called for are appropriate and meaningful for students at the level of the product and for the material under study. (p. 8)

The use of interactivity to play games such as the Tower of Hanoi was appreciated by many students. Cynthia speaks of this benefit in the following comment:

> The interactive nature of the web makes it useful in learning new concepts

because you learn hands on when you are playing a game such as "Tower of Hanoi". The interactive aspect of the internet holds the attention of the student much longer than a regular page of information such as is found in a textbook.

The Tower of Hanoi and Colouring Pascal's Triangle pages, allowed students to use interactivity to play games or perform certain activities. Another form of interactivity within the Pascal's Triangle web site, was the provision of external links that allowed students to pursue particular areas of interest. Even though the results of the navigational analysis showed that students did not take much advantage of these, a number of students still commented that they enjoyed the opportunity to explore. This is summed up by Kathee who writes: "I also enjoyed the interactive aspects of the different web sites. I enjoyed being able to explore different or related topics with the ease of clicking your mouse."

A third type of interactivity was found within the Pascal's Triangle web site. This was provided by the use of forms that allowed students to enter answers to questions. These answers could be checked by linking to a "check" page. A number of students said that they found these interactive forms and checks helpful, especially on the introductory page. This type of interactivity is described by Jane:

> For me this type of learning was very useful because it was interactive. I read about the concept, learnt how to use this concept and then answered some questions about it. It was great because I could see if I had the correct answer right away.

Variable Entry Activities and Motivation

A third reason contributing to the motivational aspect of the Tower of Hanoi site was that the game was variable entry. Variable entry is described as "tasks which are constructed so that learners, regardless of their backgrounds in mathematics, are able to locate themselves and negotiate the difficulty of the tasks they set for themselves" (Davis, 1996, p. 96). Again the interactive nature of the World Wide Web supports the design of variable entry tasks since students have control over the starting point. In the Tower of Hanoi game, students can start to play with little understanding of the mathematical concepts. As their skill and understanding increase, more disks can be added to reach higher levels of difficulty.

Variable entry is what I believe distinguishes the Tower of Hanoi from the Interactive Colouring site. In the Interactive Colouring site students were able to choose a modulus that would be used to colour the numbers in Pascal's triangle to form patterns. Even though students could choose from different modulus to make the resulting diagram as simple or complex as they liked, students still needed to be able to understand the modulus concept before they began. Feedback from students about the Colouring Pascal's Triangle site expressed frustration because students did not easily grasp the concept of modulus resulting in problems with understanding the diagrams presented. Many comments expressed opinions similar to Laurie, who wrote:

> This page explained the concept of "modulus", which I did not understand at all. I tried to grasp the concept, but after reading it over and over for 15 minutes I got frustrated and went to the bottom of the page and clicked on "probability" so that I could try a different problem.

Data Base Fascination and Motivation

The capacity of the World Wide Web to store vast volumes of information is given as a reason by one student for increasing her motivation. Betty writes: "Every web page is different; never having one that is the same. The curiosity of a user will always push him to advance to the next site because something new and interesting always pops up." Marcus (1996) discusses this phenomenon. He categorises Internet users into two categories: "browsers" are users that browse through the web looking for anything that is interesting, and "hunters" are more goal oriented users looking for specific information. He discusses the curiosity that both these groups have when using the web and states:

> Both browsers and hunters soon become familiar with a phenomenon that came to be called "data base fascination". People will sign on to a system, start retrieving information, and "suddenly" realise that a lot of time had passed. They have succumbed to the mesmerising effect of looking into (or "through") their computer screen, entering a world that consisted entirely of information. (p. 181)

This fascination may be one of the reasons a recent Strategies Group study (1998) found that more than two thirds of internet users spend less time on all leisure time and social activities now than before they used the Internet.

Multimedia and Motivation

One of the last, and possibly most important reasons, why "virtual classrooms" are good motivational learning environments for students is their ability to deliver material using a wide range of multimedia. The Pascal's Triangle site incorporated a minimal amount of these, focusing mainly on text supplemented by graphics, yet students still spoke of these advantages in their reflections. Paul wrote:

> Through my internet experience, I found that the benefits of such a mode of learning include an increased potential for visual aids. Certain people can grasp a concept much easier with images, and the computer screen offers a wide variety of stationary and animated images.... For kids particularly, the internet also offers attractive sights and sounds and a video-game-like appeal.

The motivation provided by multimedia may be even more important for today's generation of youth. A paper written by Strommen & Lincoln (1995) on how increases in technology have changed the culture of our students, discusses how important it is to adapt our teaching methods to match their experiences.

> Our children have been raised in a world of instant access to knowledge, a world where vivid images embody and supplement information formerly presented solely through text. They are used to an environment where they control information flow and access, whether through a video game controller, remote control , mouse, or touch-tone phone. (p. 1)

Summary: Motivating Students and the World Wide Web

Students found the Pascal's Triangle web site motivational in its approach to learning. This was evident in the number of students who described the site using words such as "fun", "exciting", "rewarding", "interesting", and "entertaining." This is made even more significant when combined with the fact that many of the students also expressed frustrations with the difficulty level of some of the concepts presented. Reasons contributing to the motivational aspect of the World Wide Web likely result from a number of factors, including the use of legends, interactivity, variable entry, data base fascination, and multimedia. If mathematics teachers take advantage of the opportunities provided by the World Wide Web, they will be able to make the teaching and learning of mathematics more rewarding and exciting than it traditionally has been.

Social Interaction and the World Wide Web

> Margie went to the schoolroom. It was right next to her bedroom, and the mechanical teacher was on and waited for her. It was always on every day except Saturday and Sunday, because her Mother said little girls learned better if they learned at regular hours. The screen was lit up, and it said: "Today's arithmetic lesson is on the addition of proper fractions. Please insert yesterday's homework in the proper slot." Margie did so with a sigh. She was thinking about the old schools they had when her grandfather's grandfather was a little boy. All the kids from the whole school neighbourhood came, laughing and shouting in the schoolyard, sitting together in the schoolroom, going home together at the end of the day. They learned the same thing so they could help one another on the homework and talk about it. And the teachers were people... (Asimov, 1973, p. 158)

An overwhelming number of students also missed the social interaction described by Margie in this futuristic classroom. Many students participating in the "virtual classroom" voiced concerns about the computer becoming the classroom of tomorrow and replacing the traditional classroom . Students' concerns about social interaction when using the World Wide Web centred around three issues: concerns with the lack of teacher support in general, concerns with the lack of teacher support specifically when learning mathematics, and concerns for the general lack of social interaction when using the web.

The Need for Teacher Support

Much of students' concerns about the "virtual classroom" focused on the need for teacher support. Twenty-five out of thirty-six students discussed the need for teacher support in the classroom to give them instant feedback. Many of the students echoed the opinions of Paul who wrote:

> In reflecting back on my experience as a student in elementary school and high school, my most fond and vivid memories are not of, for example, what we covered in maths, socials, or science, but rather of the special way in which the teachers taught their subjects and their genuine care and concern towards myself and my classmates, a vital component of education which a virtual classroom could never provide.

Another student, Susan, wrote about the need for teacher support to help her in understanding concepts:

> In completing this assignment I also concluded that it was difficult to learn through the "virtual classroom" and without the help of the teacher present to provide me with an alternative explanation I would have been completely lost and even more confused. Looking back on my experiences with the Internet I find it truly amazing that one can explore the world from a chair in front of a computer but for me the internet is simply a good source of ideas, entertainment, and certain information; therefore, teaching math

should be left to real life teachers who are paid to do so.

Many other students expressed opinions similar to Paul's and Susan's, reflecting their desire to keep the teacher in the classroom to provide support both for emotional reasons and for help when experiencing difficulty.

The Need for Teacher Support; Especially in Mathematics

Some students suggested that the subject being studied was a factor in determining the amount of teacher support needed when learning via the World Wide Web. Students were willing to learn from "virtual classrooms" in subject areas where they felt competent. This was expressed by the following student who wrote:

> I do not think that a person can learn about math on the Internet. A subject like History can be dealt better with the Internet, because there is no real interaction needed. As well, for someone who is lost in math the attention span can be limited and some jokes are needed.

Many students agreed with this viewpoint and said that they find mathematics in particular difficult to learn over the World Wide Web because they find mathematics more difficult then other subjects.

Students also stated that the level of mathematics being studied was a factor in determining the amount of teacher support needed. Some students, such as Sarah, said that they would feel more comfortable using the World Wide Web for lower levels of mathematics.

> I believe some of the simpler concepts in mathematics can be understood using the computer, with the assistance of a teacher. For this site I found the level of difficulty fairly straightforward, making the computer a reliable

> source. Advanced levels of mathematics are more difficult to present in a virtual classroom, and I believe the computer loses its teaching capabilities.

The Need for Social interaction in General

Besides missing the support given by a teacher in the "virtual classroom" many students missed social interaction in general. Nineteen students commented that they missed the social interaction and cooperative learning that takes place in traditional classrooms. For many, going to school teaches more than what is found in a textbook. The importance of communication skills in learning was expressed by Dave, who wrote: "The role of the teacher and other students are also vital in the social aspect of learning. Communication with other people will always be important in learning. " Another student, Kevin, wrote about his frustration with the general lack of social interaction:

> The internet is simply not the educational panacea that it is often made out to be. The lack of social contact and continuous frustration with clogged communication networks causes the internet to be much less informative than it is made out to be. The relative monotony of sitting by oneself with eyes fixated on a glowing monitor often left me feeling depressed and isolated. It would be suffice to say that the internet lacks the much needed social aspect of learning.

Summary: Social Interaction and the World Wide Web

As the students have suggested, the roles of teachers and students in the classroom go far beyond that of giver and receiver of information. Based on this research schooling as described by Isaac Asimov is still a long way in the future for most students. However, this does not mean that sites such as Pascal's Triangle can not be used within schools, only that we must pay attention to how these sites are used. If "virtual classroom" web sites are used where there is adequate teacher support and peer interaction, then the introduction of sites such as Pascal's Triangle can possibly enhance social interaction. "The World Wide Web has the potential to change the structure of the classroom, extending it beyond a literal room to include other places where students interact with other students and teachers" (Quinlan, 1997, p. 22).

Learning Mathematics and the World Wide Web

Evaluating the effect that a mode of instruction has on student learning is a difficult task. Standardised tests often lack the ability to evaluate factors such as self-confidence, communication, learning independence, higher order thinking, and originality of thought. For example, the 1996 report on the Effectiveness of Technology in Schools, commissioned by the Software Publishers Association and conducted by the Interactive Educational Systems Design consulting firm, (1996), has made conclusions about the effect of computers on both student achievement and attitudes. The report states that "educational technology has been found to have positive effects on student attitudes toward learning and on student self-concept" and that "education technology has demonstrated a significant positive effect on achievement" (p. 2). As well, an article in Time magazine discussing what makes a good school states that "as a learning tool, computers make kids adventurers and avid learners, taking them beyond the traditional walls of the schoolhouse" (Wulf, 1997, p. 65). Data from students using the Pascal's Triangle web site seemed to validate these claims.

Students' Attitudes Towards Learning Mathematics

My experience as a teacher has shown that students who enjoy school will often try harder and do better than those students who are not as motivated. Therefore, measuring the attitudes of students is an important step in evaluating how effective learning will be.

Student attitudes towards learning can be measured in a variety of ways. Most students were motivated to learn when using the Pascal's Triangle web site and this increased motivation likely leads to a more positive attitude towards learning. Offsetting this gain in positive attitude towards learning was the frustration that many students expressed because of the lack of teacher support when using the site.

The arguments for suggesting that computers increase student self-concept focus on the fact that students can remain anonymous when using the computer and wrong answers that they give are not seen by their peers (Reinhardt, 1996). While exploring this site, students were able to remain anonymous by using help pages when they experienced difficulties, however none of the students discussed this as an advantage for increasing self-concept. The issue of self-concept may have been more of a concern if students had explored the site while working in a group rather than independently.

Students' Achievement and the World Wide Web

Some insights into students' achievement can be found by reviewing the problems that students chose to work on as part of their project. The students could choose from one of three problems: the Tower of Hanoi problem, the Interactive Colouring Problem, and the Fibonacci Jigsaw. Of these three problems the Tower of Hanoi was the most accessible to students in terms of the entry level, followed by the Interactive Colouring problem, and then the Fibonacci Jigsaw.

The Tower of Hanoi Problem

Twenty-one of thirty-five students chose to answer the Tower of Hanoi problem. The instructions for this problem were: "Use the interactive site to play the game and then discuss the solutions for the minimum number of moves that can be made for games that start with 1 to 10 disks. Explain how Pascal's triangle can be used to find the solutions for the game." Eighteen of the twenty-one students who chose to work on the Tower of Hanoi problem gave solutions that were good or excellent. Most students began by describing the minimum number of moves for games using a small number of disks, and then went on to describe the patterns that they saw. Five of the eighteen students went on to complete the problem by discussing how the minimum number of disks could be determined using the various rows of Pascal's triangle. The other thirteen students extended the problem, beyond what was required, by also describing formulas which could be used to find the minimum numbers of moves. Figure 1 shows a typical answer given by one of these thirteen students.

Solution for the minimum number of moves (1-10 Disks):

# of Disks	# of Moves	Row #
1	1	Sum row 1 - 1
2	3	Sum row 2 -1
3	7	Sum row 3 -1
4	15	Sum row 4 -1
5	31	Sum row 5 -1
6	63	Sum row 6 -1
7	127	Sum row 7 -1
8	255	Sum row 8 -1
9	511	Sum row 9 -1
10	1023	Sum row 10 -1

1. Solution using Pascal's Triangle: Correspond the # of discs used in the game to the row # in Pascal's Triangle. Take the total of the numbers in that row and subtract 1. It will equal the minimum number of moves necessary to play the game. (when numbering the rows in Pascal's Triangle start with the first row being row 0 and then continue downwards)

Figure 1. A typical answer to solving the Tower of Hanoi problem.

2. <u>Recognising the Pattern:</u>

# of Disks	# of Moves	Formula: (a=2) (d = 2) (q = 2)	
1	1	$a_1=a_1$	$a - 1 = 1$
2	3	$a_1+d = a_2$	$(a \times q) -1 = 3$
3	7	$a_2+d^2 = a_3$	$(a \times q^2) -1 = 7$
4	15	$a_3+d^3 = a_4$	$(a \times q^3) -1 = 15$
5	31	$a_4+d^4 = a_5$	$(a \times q^4) -1 = 31$
6	63	$a_5+d^5=a_6$	$(a \times q^5) -1 = 63$
7	127	$a_6+d^6=a_7$	$(a \times q^6) -1 = 127$
8	255	$a_7+d^7=a_8$	$(a \times q^7) -1 = 255$
9	511	$a_8+d^8=a_9$	$(a \times q^8) -1 = 511$
10	1023	$a_9+d^9=a_{10}$	$(a \times q^9) -1 = 1023$

Equation to find the minimum number of moves required for a certain number of discs:

$M = (a \times q^{n-1}) - 1$

M= the number of moves

n= the number of discs

$a = 2$

$q = 2$

Figure 1 continued.

The three students that did not give good or excellent solutions to the Tower of Hanoi problem described the minimum number of moves needed to play the game and gave a formula for finding the minimum number of moves, but they did not describe how Pascal's triangle could be used to solve the problem.

Help pages were available to students working on the Tower of Hanoi problem, but the answers I received from the students were not plagiarised and students demonstrated they had put thought into providing their own answers. Overall, students seemed to be successful in solving the Tower of Hanoi problem, demonstrating that they had a solid grasp of both the problem and the solutions. As well, it seemed that students were able to find more than one way to solve the problem, reflecting the greater access to ideas and information found on the Web than in a traditional environment.

The Interactive Colouring Problem

Of the three problems that students could choose to solve, the Interactive Colouring problem was the second most popular choice. Thirteen of thirty-five students chose to work on this problem. The instructions given to students for this problem read: "You should discuss what the term 'modulus' means, how it is used to colour the triangle, and similarities and differences found in the patterns that are produced." Of the thirteen responses to the problem, only four students gave complete solutions. These students described the concept of modulus, described how different modulus could be used to colour the triangle, and then went on to describe patterns they observed when using the different modulus. Some of the patterns that were described related to symmetry, prime numbers, the direction in which triangles point, and the number of lines in each modulus before multiples are found. Six students were able to give a satisfactory explanation of the modulus concept and to illustrate how modulus can be used to colour the triangle, but they did not discuss any patterns that were found in the triangles. When explaining the concept of modulus, most students gave explanations similar to the one given on the site which relates modulus to division and gives an example of how to determine remainders using groupings of sticks. One student in this group of six was unhappy with this explanation of modulus and gave an excellent analogy using pieces of lumber for construction. Figure 2 shows a typical answer given by one of these six students.

The remaining four students that worked on the Colouring Pascal's Triangle problem were able to describe the concept of modulus but were unable to give satisfactory explanations of how modulus can be used to colour Pascal's triangle.

Students' achievement level on the Colouring Pascal's triangle problem was noticeably lower than on the Tower of Hanoi problem. I believe students were initially attracted to this problem because of the interactive colouring, but they became frustrated and confused when trying to understand how modulus was used to colour Pascal's triangle. My observations during tutorial sessions showed that students had difficulty with the concept of modulus, especially when the number being coloured was smaller than the modulus, such as 3 modulus 4. The lack of teacher support to give instant feedback to students when they encountered these problems, was given by many students as the cause for their lower levels of success.

Colouring Pascal's Triangle: A Solution

After puzzling over this site for quite some time I came to finally understand the nature of this problem. The key lies in the definition of 'modulus'. As explained in the site, modulus is very much like division. If we take 7 sticks and put them into groups of two we have 7 modulus 2 with 1 stick remaining. This is important to understand because this is how the triangles are formed and coloured.

Using the interactive tool to alter the row size, modulus or size of the triangle, I discovered many different varieties of triangles. One thing many of them had in common was an outline coloured red. I began to understand how the triangle was being colour coded! The outside numbers in Pascal's triangle are always 1. So, with this in mind I figured

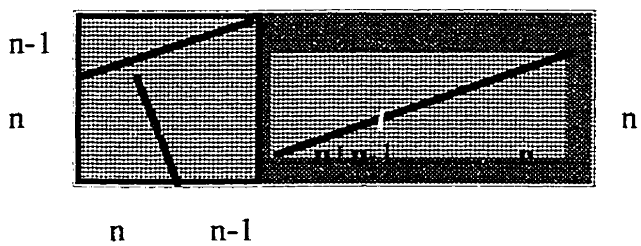
out that each colour represents a remainder such as 1 and this is given a colour. After matching the modulus to the triangles and then checking my answer, I discovered this was correct. A remainder of 0 is coloured black, remainder two is coloured green and so on. Colours are given arbitrarily but the same numbers are always the same colour and the colour is given to no other. The higher the modulus, the more colours are represented. Also, the number of different colours is directly proportional to the modulus. For example, modulus four would have four colours. More remainders are evident with greater variety of colour. As a result, the higher the modulus, the more complicated the pattern and the more colours displayed.

Figure 2. A typical answer to the Colouring Pascal's Triangle Problem

The Fibonacci Jigsaw Puzzle

The third problem, the Fibonacci jigsaw puzzle, was the least popular with the students. Only two individuals chose to work on this problem. The instructions asked students to "Print out the page that is provided and try the puzzle. Discuss whether this puzzle will work with other numbers in the Fibonacci sequence and why the areas of the starting square and resulting rectangle are not the same." Of the two students that undertook this problem one student was able to complete the exercise. Dan used trigonometry to show that the dimensions of the rectangle are not accurate. He then completed the problem by showing algebraically that this puzzle works for all Fibonacci numbers. Figure 3 shows Dan's answer to the Fibonacci problem.

Let "n-1" and "n" be any number in the Fibonacci sequence where (n > n-1), then the resulting square and rectangle will be as follow.



Area of square=(n+n-1)(n+n-1)
=(2n - 1)(2n - 1)
$= 4n^2 - 4n + 1$

Area of rectangle=(n+n+n-1)(n)
=(3n - 1)(n)
$=3n^2 - n$

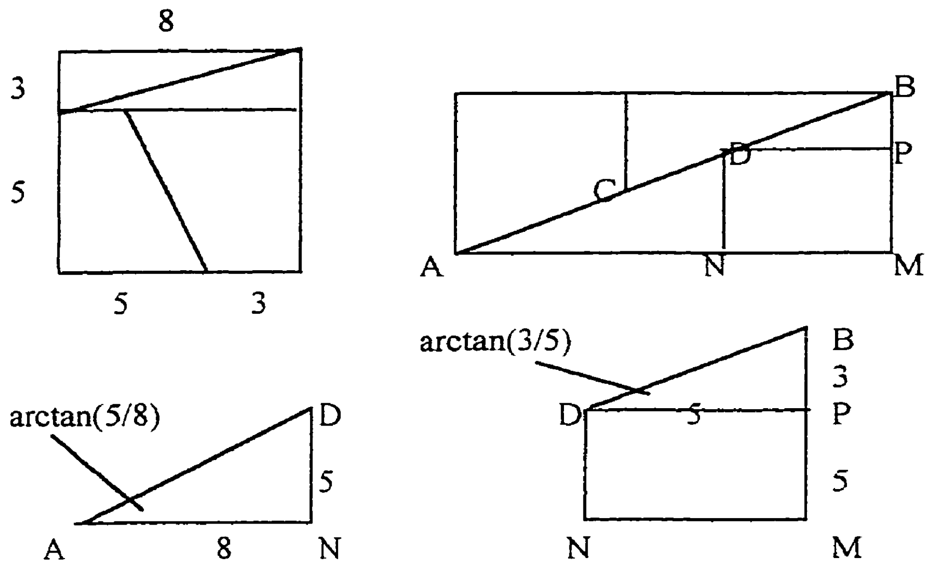
Figure 3. Dan's solution to the Fibonacci Jigsaw Puzzle

	$4n^2 - 4n + 1$	$3n^2 - n$
n=0	1	0
n=1	1	2
n=2	9	10
n=3	25	24
n=4	49	44

when $n = 0$ and $n \geq 3$, area of square > area of rectangle.
when $1 \leq n \leq 2$, area of square < area of rectangle.

However, it can't be true, because how can a square unit be created or destroyed. There must be some dimension error or some lines are not straight as it seems.

Lets analyse 8 x 8 square with the 5 x 13 rectangle:



Since DP is parallel with AN and if AD and BD are on a straight line then angle DAN should equal angle BDP, however that is not the case, therefore AD and BD are not on a straight line and therefore there are holes or gaps in the middle of the (so-called) rectangle which contribute to the difference of area. Since AB is not a straight line, then its not the diagonal of the rectangle, and therefore it is not a rectangle.

Figure 3 continued.

The second student's explanation for the Fibonacci problem did not include a solution for why the areas of the square and rectangle are not equal. However, she did use a unique approach to the problem. This student used previous knowledge which she had learnt in class about the number of squares crossed by diagonals in a rectangle. Using this knowledge, she showed that the rectangle must be missing one square unit, but she failed to explain how this could be true. This student's approach confirms the belief of constructivists, that when students are given choices in their learning, they will chose activities that will build on their preexisting notions.

Summary: Learning Mathematics and the World Wide Web

The fact that many students find learning via the World Wide Web motivational results in some added benefits for learning mathematics. Those students that enjoy learning are likely to do better in school. The following excerpt from the "Report on the Effectiveness of Technology in Schools, 95-96" (Interactive Educational Systems Design, Inc., 1996) confirms that the increased motivation which results from using computers, does have an impact on student attitudes and self-concept.

> Educational technology has been found to have positive effects on student attitudes toward learning and on student self-concept. Students felt more successful in school, were more motivated to learn and had increased self-confidence and self-esteem when using computer-based instruction. This was particularly true when the technology allowed learners to control their own learning. (p. 2)

This study also confirmed that students are able to reach satisfactory levels of achievement when learning mathematics over the World Wide Web. In analysing students' answers to the three problems, it seemed that most students had a solid base of understanding and many students went beyond what was required. The unique approach to the problems offered by many students suggests that this mode of learning also allowed students to be more adventurous than they would be in traditional classrooms.

The problem that caused students the most difficulty involved using modular arithmetic to colour Pascal's triangle. I believe that this was caused by two factors. First, the site presented students with a poor explanation for the concept of modulus and second, the students had difficulties with understanding the concept of remainder when the dividend was smaller than the divisor. Support for the students either in the form of teacher presence, on-line support, or group work seems necessary when working on these more difficult problems.

Solving problems over the World Wide Web offered students the chance to access a vast resource of information to aid them in discovering a solution. Students gave evidence of this in answers that presented more that one method for solving a problem. The World Wide Web allowed students to view a variety of solutions and shifted the emphasis from finding an answer to discovering a process.

The Content of Web Pages

The World Wide Web has many advantages as a medium for instruction. However, the medium itself is not enough to ensure effective learning for students: one must also focus on the content within the pages of the site.

> Developers need to recognise that bells and whistles are only impressive when they are attached to an impressive engine. Content instruction is the engine of education. Concentrate on the content; make it solid; provide powerful tools to teach it well. Then instructional hypermedia/multimedia products can realise their promise. (Cates, 1992, p. 9)

In exploring a variety of mathematics sites on the World Wide Web I have found only a few that are pedagogically appropriate for junior high school students. Sites are often unsuitable in the level of information presented, the format of the presentation, the use of inappropriate links, and the lack of interactivity available for students.

In observing students at work on the Pascal's Triangle web site, I found that the content of each page had a large impact on students' interest level, attention spans, and learning. This was also indicated by the comments that students wrote in their journals as they progressed through the variety of pages within the site.

Pascal's Triangle Page

The first page of the site to which students were directed to was the page entitled Pascal's Triangle. The purpose of this page was to introduce students to Pascal's triangle and show students how the triangle could be generated, since this knowledge was a prerequisite for pages to follow. The majority of students commented that this page was easy to understand. Students were able to check their understanding of how to generate Pascal's triangle by entering numbers directly into a triangle found on the page and then linking to a page to check their input.

The feature that made this page unique was the interactivity that allowed students to check their answer. This enabled students to interact in a way that was meaningful rather than just using interactivity to turn pages. Many students commented that this interactive check was helpful in aiding their understanding. However, this check would have been more efficient if the numbers that students put into the triangle could automatically be checked without students having to link to another page. This can likely be accomplished and is a feature that I intend to investigate for the future.

Three students also commented in their double-entry journals that they found the information contained within the Pascal's Triangle page interesting because it discussed the historical background of the triangle and its namesake.

Colouring Pascal's Triangle Page

The second page to which students were directed was the Colouring Pascal's triangle page. The purpose of this page was to motivate students and to allow them to search for patterns that can be found in the triangle. The two participants in the pilot study enjoyed using the interactive colouring page and were impressed with the images that were produced. However, when this page was used with the MATH 190 students many of them found the experience frustrating and confusing. This was probably in part due to the different expectations that were placed on these students. They were asked not only to explore the site, but they were also given the task of completing one of three math problems, a choice which included the Colouring Pascal's triangle problem. Because of this I believe their approach was different. Rather than using the interactive tool to "play" and observe patterns, they focussed more on the concept of modulus and its meaning. This is what I believe led to their frustration, since many students commented that they had a difficult time understanding this concept and its application in colouring Pascal's triangle. The explanation given for modulus on this page was that "modulus was similar to division" and an example of how to find modulus was illustrated using an analogy which involved the grouping of sticks.

A much better explanation of the modulus concept was given by Carl who used a construction analogy to explain it and in redesigning this page I would include his analogy.

> If a prefabricated wall comes in 8-foot sections the builder speaks of this as a module and seeks, for the various elements of the construction, lengths that will fit the 8-foot module. The architect plans the lengths of all his walls, wherever possible, to come out in multiples of 8. Tiles, windows etc. can be assembled with the least amount of trouble if they fit the module.
>
> However, it may not be possible to build the whole house this way. Suppose the carpenter looks at the plans and sees a length of 11-foot wall, and at another place he finds a 19-foot wall. He notes that one 8-foot section will leave 3 feet over for the 11-foot wall, and two 8-foot sections can be used for the 19-foot span, but likewise will leave 3 feet unaccounted for. Therefore, the problem to be dealt with in both cases is the same. It makes no difference how many modular sections are to be used; that's the easy part. What must be filled in with some kind of hard work is the space of three feet- the remainder on division by 9. The fact

that the carpenter has the identical problem in both cases is expressed in number-theoretic language by saying that 11 and 19 are congruent modulo 8.

Another problem that students encountered when using the Colouring Pascal's triangle page was with the "test your skill" question provided at the bottom of the page. Students were asked to identify what modulus was used to colour six different triangles shown. While this "test your skill" section was successful on the first page, it did not seem to be so here since students became confused and frustrated when they could not come up with the correct answers. As one student commented "I found this difficult because the triangles were a small image size and by this point I still hadn't understood the correct way to use Pascal's triangle. It was too early to be tested".

A review of this page revealed three features that were important to the content of web pages. The first was that the approach taken must be appropriate for the content of the page. This type of interactivity would have been more successful if students were first given the opportunity to "play". Then students could have been given more structured guidance to aid them in understanding the concept of modulus that underlies the colouring of the triangle.

The second feature revealed from the analysis of this page was the importance of the wording and analogies used when explaining new concepts to students. The concept of modulus was very difficult for students to understand and they required more detailed instructions and a better analogy to aid them in their comprehension.

The third and last feature revealed by this page relates to "test your skill" interfaces. These interfaces should have been a last review for students when their chance of having some success was high. Placing these in the page before students have a good grasp of the concept only contributed to their frustration.

Chinese Triangle Page

After the Pascal's triangle page and the Colouring Pascal's triangle page, students could make choices in their navigation. The choices that students made were varied which indicated that different topics appealed to different students. All of the pages were described as being interesting by at least one student. However, of the four remaining pages, the Chinese triangle page had the fewest number of students commenting that they found it interesting or engaging. The purpose of this page was to expand on the historical background behind Pascal's triangle and to allow students with an interest in Chinese mathematics to learn more on this topic. This page was one of the shortest in the site. It gave a short background on the Chinese triangle and it included a diagram showing this triangle using numbers written with ancient Chinese symbols. A task invited students to use the symbols written in the triangle to determine the Chinese symbols for the numbers 8, 11, 22, and 35. Some students attempted this question, but none reported that they were successful with many students citing the poor quality of the diagram as a problem. This page ended with links to one site that gave the history of Chinese mathematics and a second site that gave a detailed account of the abacus. Students who visited the abacus site stated that they found it interesting.

The Chinese Triangle page illustrates some undesirable design features for educational web pages. Web pages need to contain enough information to hold students' attention. Students commented that they enjoyed the historical information presented, but more information was needed regarding the Chinese and their development of the triangle.

The second design feature to avoid is poor graphic quality when posting images on a Web site. The image of the Chinese triangle was scanned into the Web file and better equipment along with greater expertise should have been used to improve the quality of the graphic.

The third mistake found in this page involved the "skill testing question" that was given for students to answer at the bottom of the page. Two problems existed with this question. First, it was very difficult to solve this question using the information given. The question needed to be rewritten allowing students to go through the process of finding patterns. Second, the question itself was not relevant to the generation and understanding of the triangle, and a problem involving the patterns within the triangle would have been better suited.

Pascal and Probability Page

According to the students, the next least interesting page was titled Pascal and Probability. This page discussed how Pascal further developed his triangle for applications involving probability. Students were given an example of how the triangle is used to solve a probability problem involving coin tossing. Students were then given a probability question to solve which involved finding partners for a doubles tennis match. Students were able to link to a check page that explained how to find a solution to the problem. After this, students were introduced to the Tower of Hanoi problem and were given the option of linking to the University of Toronto site to play the game. Only four students commented that they found the Pascal and Probability page interesting, however, eight students commented that they were interested in the Tower of Hanoi game that this page was linked to.

The main drawback to the Pascal and Probability page was the difficulty level of the problems presented. Students seemed to be confused with the problems and explanations that were on this page. Nine students commented that they found the page confusing and difficult. For these more difficult problems it would be better to put fewer problems on each page and to expand on the problem. Students needed to be given more guidance and suggestions for possible methods that can be used in problem solving.

Blaise Pascal Page

A page that students found more interesting was the one describing the life of Blaise Pascal. This page gave an overview of Blaise Pascal's life including his work in mathematics and physics, and his dedication to religion. Many students commented that they found learning about the backgrounds of mathematicians they are studying interesting. One wrote: "This site was really interesting. Math is not just about numbers and calculations. Learning about the lives of people who came up or discovered these theories and so forth, is indeed really interesting." This page also gave links to two other sites on Blaise Pascal and to a third site that discussed the calculator which Blaise Pascal developed.

The Blaise Pascal page illustrates the interest that many students have in learning about the personalities behind the mathematicians they are studying. Blaise Pascal's religious beliefs and family traits makes the mathematics more interesting for students to learn.

Pascal and the Fibonacci Sequence Page

The last topic that students could explore in the Pascal's triangle web site was the discussion of Pascal and the Fibonacci sequence. There was an abundant amount of information that I wanted to present on this topic, so rather than presenting it as one very long page I split it into two pages. The first page was titled Pascal and the Fibonacci sequence and the objective was to show students that yet another pattern existed in Pascal's triangle. This page starts with a history of Leonardo da Pisa (Fibonacci) and how he generated the sequence of numbers called the Fibonacci sequence. It then uses a graphic to show students how to find the Fibonacci sequence in Pascal's triangle. Lastly, the relationships between the Fibonacci sequence and the Golden Ratio, and between the Fibonacci sequence and patterns in nature, are introduced. Six students commented that they found this page interesting. Students commented that they were surprised to find the Fibonacci sequence in both Pascal's triangle and in nature. One student wrote: "I found this page VERY interesting. At first I could not see the Fibonacci sequence in Pascal's triangle but after a while of looking at it I figured it out. I find this amazing!"

Besides commenting that the Pascal and Fibonacci page was interesting, a number of students said that they had difficulty understanding the information presented on this page. In particular students could not see how the Fibonacci sequence could be found in the triangle.

The Pascal and Fibonacci sequence page was successful in its attempt to show students that there are many relationships between the Fibonacci sequence and other aspects of mathematics. However, a lot of information was presented on this page and this likely caused a number of students to find it difficult. Some information could be eliminated or presented on a separate page.

Fibonacci Investigations Page

The page on Pascal and the Fibonacci sequence was linked to a second page titled Fibonacci Investigations. This page gave students three problems to solve involving the Fibonacci sequence and linked to a external site on the Fibonacci sequence. Students found this page more interesting than other pages in the site. Nine students used words such as "interesting" and "engaging" when describing this page. Students gave a variety of reasons for liking this page. Among them were the many applications of the Fibonacci sequence, the surprising nature of the Fibonacci puzzle, and the relationship between the Fibonacci sequence and nature. Again, a number of students described this page as being difficult, but they did not express the same frustrations with the activities as they did on the Colouring Pascal's triangle page. I believe this may be because all of the problems presented on the Fibonacci Investigations page could be explored without necessarily having to arrive at solutions .

The success of the Fibonacci Investigations probably lies in the variety and novelty of the information presented. Many students found the mathematics "surprising" and there was a number of different problems to solve. Check pages were provided that gave students explanations for the problems when they ran into difficulty, relieving some of the frustration that students felt. As well, the activity involving the Fibonacci jigsaw puzzle was presented on a page that could be printed and gave students an opportunity to explore "hands-on" solutions.

Summary: The Content of Web Pages

The content contained within each Web page had an impact on students' interest level and motivation. Some common threads could be seen in the pages and activities that students found interesting. First, the addition of interactivity within the page seemed to elevate the student's interest level. Students definitely enjoyed playing the Tower of Hanoi game and even commented that they enjoyed the interactive colouring in spite of the fact that they found it confusing. Second, many students enjoyed the historical background that was presented on the Blaise Pascal page. A third element that seemed to add interest to pages was the addition of problems, as long as they were presented in a way that allowed students to achieve some success. More difficult problems needed to be restructured using a series of steps to help guide students towards a solution. Some pages could have become more approachable for students if better graphics and clearer instructions were given. The last element that seemed to make a page more interesting was the amount of variety found within the page. Pages such as the Chinese triangle page were too brief and needed an activity or extension to make the page more appealing.

CHAPTER FIVE

DISCUSSION AND CONCLUSION

This discussion will review the lessons which have been learned as a result of completing this research. I will discuss the possible applications of the Pascal's Triangle web site and how these applications may affect students' learning. The pedagogies of teachers are likely to be affected when using this site and these pedagogies will also be a focus of the discussion. This will be followed by a discussion of the five main themes that were identified through this research and a summary of what was learned about each theme. In concluding the discussion, I will analyse whether this research has accomplished its goals, identify what questions are still left to be answered, and make some suggestions for further research.

Using Web Sites within the Classroom

In this study students used the Pascal's Triangle web site in two different ways. The two Grade 9 students in the pilot study were asked to simply explore the site for approximately one hour and then give feedback. The students in the MATH 190 course had much more specific instructions about how to use the site. These instructions were divided into three categories: keeping a journal, solving a mathematical problem, and writing their reflections. Both of these approaches to using the site had their advantages and disadvantages and either approach would probably appeal to some teachers depending on their teaching styles.

Approaches for Using "Virtual Classrooms" and their Impact on Student Learning

Both of the Grade 9 students that used the site in the pilot study said that they enjoyed the experience and would like to use a site such as this to learn mathematics in their school. Both students gave feedback about the design of the site and problems they encountered as they surfed through the site. They commented that for some pages they required more background information than they had acquired and that some explanations were poor. However, these students did not seem to become "bogged down" when they encountered poor explanations, instead they moved on to another page or activity. The knowledge that these students gained was a more general sense of Pascal and his triangle rather than a focus on specific problems and activities. These students seemed to "play" with the site more than the MATH 190 students and enjoyed activities such as the interactive colouring of Pascal's triangle. When asked about how teachers should approach the use of this site in the classroom, one student suggested that a more structured approach was needed and the other student thought that a supporting worksheet might be a good idea.

Students in the MATH 190 class had quite a different experience than did the Grade 9 students in the pilot study. Most students seemed to enjoy using the site, although they gave more specific and critical feedback. Students became "bogged down" when they encountered problems and many expressed frustration as a result of this. Some students commented that when they had difficulties with understanding a particular problem they would move on to the next page or activity, but overall I think these students put more effort into solving the problems. The knowledge gained by these students seemed to reach greater depths than was reached by the two Grade 9 students. Some of the MATH 190 students commented that having to write down their travels in a journal slowed them down and that they would have preferred having the opportunity to "surf" the site without having to keep a journal.

The two different approaches to using the Pascal's Triangle web site both have strengths and weaknesses. If the goal behind using a "virtual classroom" is mainly to motivate the students then a less structured approach is suggested. However, if the goal is to focus on specific learning outcomes then a structured approach should be used.

Approaches for Using "Virtual Classrooms" and their Impact on Teacher Pedagogies

When using a "virtual classroom", I think teachers have a number of options in terms of their approach. The approach that teachers take will depend partly on their teaching styles and partly on their desired learning outcomes. An unstructured approach will appeal to teachers with a constructivist pedagogy who believe that students should be offered choice in their learning. The structure of the site itself already imposes some restrictions on the students' freedom to explore, however I believe some of this is necessary to avoid becoming tangled in the web of information available. A more structured approach is better suited to teachers with more traditional beliefs who may fear the outcomes associated with setting students free to explore the web. This approach also can be used to focus on particular topics within the site allowing students to develop knowledge about a specific subject.

Most teachers have a strong commitment to ensuring that their students acquire the requisite knowledge prescribed for each course. Therefore, the course curriculum will influence teachers' decisions about whether to use a "virtual classroom" and the approach that should be taken. In British Columbia, the junior mathematics curriculum specifically calls for the use of the Internet to explore a variety of research topics. However, I think teachers would be more likely to use a web site if they were provided with support material that outlines the learning outcomes that can be accomplished by using the site and how these learning outcomes related to their particular curriculum. For example, the interactive colouring site asks students to search for patterns which relate to a number of topics in the British Columbia mathematics curricula for junior high school.

Teachers also need support in realising the variety of ways in which a site such as Pascal's Triangle can be used in a classroom. As Zhao (1998) suggests in his article on adopting new technologies into education, too often teachers are presented with a technology with the idea that "if you purchase it they will use it" (p. 307). Zhao points out that most workshops deal with teaching educators about the technology rather than focusing on how these technologies can be used to improve teaching and learning. He believes an alternative and more effective approach would be to develop applications that support a wide range of pedagogical beliefs, from traditional teacher-centred learning to more learner-centred constructivist pedagogies. I support this belief and therefore, consider it important that teachers use sites such as Pascal's triangle in a variety of ways to support their teaching styles and the specific learning outcomes that are required.

Summarising the Results of the Research

The purposes for doing research in mathematics education are manifold

(Kilpatrick, 1992). More traditional aims follow a scientific approach of trying to attempt, predict or control, while newer approaches may look at the meanings that the learning of mathematics have for the students engaged in the process. This research follows yet a third approach, where research is designed to improve practise and to allow students to have a voice in the direction for improvement.

While research does follow a set of guiding principles I believe, as Romberg (1992) suggests, that doing research "embodies more characteristics of a craft than of a purely technical discipline" (p. 51). The interviews, observations and projects that were analysed for this research suggest that there are certain trends or themes that emerge when students are involved in learning mathematics through the medium of a "virtual classroom" such as the Pascal's triangle site. These themes help to give a general sense of factors that might be important in designing, creating and implementing web sites into the classroom. However, unlike scientific experiments, the data can only lead us in a direction which will need further investigation in order to evaluate its validity.

Navigating Pascal's Triangle Web Site

The process that a student uses when learning mathematics is extremely important and I believe this process is most successful when students have some choice in choosing the path for their learning. Davis (1996) discusses a similar notion, that of "currere." Currere refers to the "running of the course" rather than the "course to be run" (p. 90). He suggests that emphasis should be placed on the path that students take when learning and that students need the opportunity to make choices about which path to take. Using a "virtual classroom" seems to lead naturally to placing more emphasis on the "running of the course", since decisions about the direction of learning have to be made every time a student is faced with choices of links to follow.

This study suggests that the way in which students chose links to follow, is different when using a "virtual classroom" than when using a "virtual library." When using a "virtual library", students seem to do more "surfing", looking for information rather than actual studying of information presented. In the "virtual classroom", "surfing" is kept to a minimum with few students visiting more than one or two external sites. This aids in focussing students' attention on understanding the information presented but it also may restrict some of the freedom that students formerly had.

Even though students were selective in the number of sites they visited, they still seemed to make choices as was evident by the variation in students' navigational paths. Comments in students' journals showed that different students were attracted to different pages. It does, however, seem possible to direct students' movement through the pages when necessary, since most students visited the first two pages before going on to other pages in the site.

Motivation and the World Wide Web

The ability to become involved in choosing the path of their learning may be one of the factors that produced increased motivation for students when using the Pascal's Triangle web site. Interactivity also seemed to be a factor in increasing motivation. The content placed on each page can also produce increased or decreased motivation. The lack of information on the Chinese Triangle page was responsible for decreased motivation while historical information, such as was found of the Blaise Pascal page, increased motivation.

Social Interaction and the World Wide Web

There was an overwhelming cry from the students to have support in the classroom for challenging situations. Students felt that teachers can supply help to meet individual needs and skills that cannot be replaced by the computer.

Students also missed the support that they receive from their peers when working on problems. They missed the social interaction that goes on in traditional classrooms and very few students seemed to embrace the idea of a "virtual classroom" completely replacing the traditional classroom.

Learning Mathematics and the World Wide Web

Perhaps, the hardest question to answer in doing research is the one which asks "Is the teaching and learning of mathematics improved by using this method?". Students exhibited their learning in the answers to the problems that they could choose to work on. It is difficult to say whether the learning was improved by using the Pascal's triangle site, but there does seem to be some differences from more traditional forms of learning. Often, students would give more than one possible solution to a question which goes against the old notion that there is only one right answer for every question. This likely was influenced by the resources that could be accessed via the links provided within the site.

Another difference in learning was that many students experienced a level of frustration before arriving at answers, and they spent time working through to a solution. Again, this seems to differ from the approach often taken in math textbooks where students are faced with a number of different problems and believe they are expected to achieve an answer within a minimal time frame.

Content of Web Pages

Some general comments regarding what content should be placed on web pages can also be made based on this research. Content which included historical backgrounds interested many of the students and gave them a sense of the personalities behind the mathematics. Interactivity was important to include especially when it allowed students to control features of a game or to check solutions to questions. The amount of content on a page was also important. Too much information was overwhelming and caused frustration, whereas too little information was boring and didn't maintain the students' interest. More difficult mathematical concepts needed to be supported with suggestions or guidance. The expectations of the teacher determined whether students felt that they could "play" with a problem or whether they needed to find the "answer."

Research Goals

The goals for this research have continued to develop along with the different stages of research. The initial goal was concerned with design issues of the site such as colour, typography, page layouts, and issues dealing with links to other pages. Some of these issues were addressed by the two students, Jack and Zachary in the pilot study. Their feedback suggested that changes needed to be made regarding the navigational paths that students followed and the amount of interactivity within the site.

The first major revision to the site design involved an attempt to lead students through the first two introductory pages before they visited other pages within the site. The redesign was fairly successful in that the majority of the MATH 190 students did visit the introductory pages before proceeding on to the rest of the site.

The second revision to the site design was to increase the amount of interactivity within the site. This interactivity went beyond simple page turning and allowed students to play games and use forms to check answers.

A second goal for this research was to focus on the themes that emerged in order to get a general sense of what implications using the World Wide Web has for the learning of mathematics. This goal was met in that five major themes were identified. These themes provide good starting points for future research that can now focus on one of the themes and explore it in greater depth.

The last goal for this research was to connect the issues of site design with the results obtained from the students' reactions and reflections in order to make more general recommendations for the design of "virtual classrooms". Some recommendations have been made by analysing the content found on the various web pages but because this site is still in the early stages of development the number of recommendations that can be made is limited. The cyclic nature of product development allows the recommendations to continue to grow and develop as further changes are made to the site design.

Future Questions and Answers

As with all innovations, there still are many roadblocks in the way before a site such as Pascal's Triangle becomes common in the classroom. As teachers and students continue to use Web resources, more will be learned about the effective design and implementation of sites. This research points to some trends that seem to arise from using the World Wide Web in mathematics education, but these trends may not hold true in other situations. The research did not take place in a traditional classroom since it was an independent project that students could do on their own time. As well, the students in the MATH 190 group were older than the intended audience for this site which was Grade 8-10 mathematics students.

Researchers in the future need to explore many issues dealing with the use of the World Wide Web in classrooms. The first of these is the continued exploration of the variety of ways in which this new technology can be applied. The "virtual classroom" is an innovative approach and there are many ways in which this concept can be expanded. The use of chat rooms, e-mail, bulletin boards, and other features of the Internet, were largely untapped in this research and many of these could solve problems that students found with the lack of social interaction. I believe that there is much potential to improve the "virtual classroom" if these features were added.

Allowing students to be involved, in a meaningful way, was an issue that this research tried to solve by adding interactivity. New scripting languages such as Java offer the programmer more exciting solutions for creating interactivity and involving students. Yet, these new languages also require computers with more advanced technology and this could render many computers obsolete. Researchers need to find ways to strike a balance between including these exciting features into web page design with the need to provide access for as many teachers and students as possible.

Student motivation was increased when using the World Wide Web. Yet, researchers still struggle to understand whether this is a result of content or the use of the technology itself. What makes one web page motivating and another not? Some answers have been suggested by this research but many still need to be validated.

The primary goal for using this web site in the classroom will be to teach students mathematics. Is the learning which occurs when using this site different from the learning which occurs in a more traditional classroom? Can the "virtual classroom" replace the traditional classroom, or should it be combined with the traditional in order to maintain a variety of pedagogies?

Lastly, researchers need to focus on the issue of teacher pedagogies and how these may be affected by the use of the World Wide Web. Can the web be adapted to suit a variety of teaching styles or must teachers adopt the constructivist viewpoint if they wish to adopt the web into their classrooms?

Directions for the Future

I am excited about the possibilities for the "virtual classroom" in the teaching and learning of mathematics. Mathematics has traditionally been taught in a style that focuses on the rote learning of axioms and on finding the "right" answer. I believe that introducing the World Wide Web into the classroom will introduce students to another world of mathematics and motivate their learning.

As past experience has shown, the implementation of new technologies is not an easy enterprise. Houston (1998) believes that inertia is the greatest barrier to innovation and that change requires effort, work and time. We need to explore more alternative methods for implementing the Internet into our classrooms and to continue to look at options such as the "virtual classroom." Teachers are just beginning to realise the potential of the Internet and the World Wide Web and many have not had enough experience to realise its full potential.

The World Wide Web has the potential to change the structure of the classroom, extending it beyond a literal room to include other places where students interact with other students and teachers. The Web also has the potential to change the structure of the curriculum, extending it beyond a textbook with answers and enriching it with Web sites that raise many questions. With the potential for students to explore topics so extensively, the teacher must become the navigator of student learning, leading students through the bits and pieces of information that students combine into unique, ever-changing images and transform these into a new kaleidoscope of knowledge (Quinlan, 1997, p. 22).

The "virtual classroom" introduces an innovative approach for teachers to use in their classrooms. The adoption of this technology not only promises to create a new kaleidoscope of knowledge for students, but also for teachers. Combining the "virtual classroom" with the compassion and understanding of teachers, promises an exciting new direction for the future of mathematics education.

APPENDIX A

Pascal's Triangle Web Site

http://ted.educ.sfu.ca/people/students/jane/

Pascal's Triangle

What is Pascal's Triangle?

Pascal's triangle is a special series of numbers organized into a triangular pattern.

Who Discovered the Triangle?

Pascal's triangle has existed for many years going as far back as 1303 A.D. when it appeared in a book written by the Chinese. Blaise Pascal further developed the use of the triangle and because of this the triangle is now named after him.

How to Make the Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

To make Pascal's triangle you need to start with the number 1 at the top of the triangle. This is the first row. The rows below all start and end with the number 1. The numbers in the middle of each row are obtained by adding the number ***directly*** to the left and right in the row above. For example, the 2 in the third row comes from adding 1+1 found in the second row above. The 3 in the fourth row comes from adding 1+2 in the third row above.

Test Your Skill

Now that you have read how to make Pascal's triangle you can test your skill. In the space below complete the next 5 rows of Pascal's triangle so that you end up with a triangle that is 10 rows in length. When you are finished click on the button below to check your answer.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Reset

Coloring Pascal's Triangle | More Pascal's Triangle | Blaise Pascal | Chinese Triangle | Fibonacci | Probability

Please E-mail comments to:
mmoor@sfu.ca

The First 10 Rows of Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

Coloring Pascal's Triangle

Interactive Coloring

One way in which patterns in Pascal's triangle can be found is by coloring the numbers in the triangle according to their divisibility or modulus. For example we could color all numbers in the triangle divisble by 2 black and all other numbers red to create the triangle you saw on the first page.

"Modulus" is similar to division. To understand modulus picture the number 7 represented by seven sticks laid out in front of you. To find modulus 2 you would group the sticks into pairs until you had 1 remaining stick. In our coloring example 7 modulus 2 would then be colored red since there is one stick left over. To find 7 modulus 3 you would repeat the same procedure except that you would group the sticks in threes rather than in pairs. This again would leave one stick remaining.

You can do this quickly on the computer by using the interactive tool below. Enter the number you wish to divide by in the space labelled "modulus". You can also control the number of rows you wish to see and the image size. (Submitting these values will take you to another site and you will have to use the "back" or "go" feature at the top of the menu bar to return to this page "Coloring Pascal's Triangle")

Select values for the number of rows, modulus, and the size of the image, and then *submit*. Otherwise the default values should generate an interesting image.

Output Image Rows (max 100): 15 Modulus (2 to 16): 8 Image size: 300

Submit Set to default

Test Your Skill

The following triangles have been colored by using the interactive coloring tool you used above. Modulus 2,3,4,5,7 and 9 were used. Can you match each of the triangles with the correct modulus?

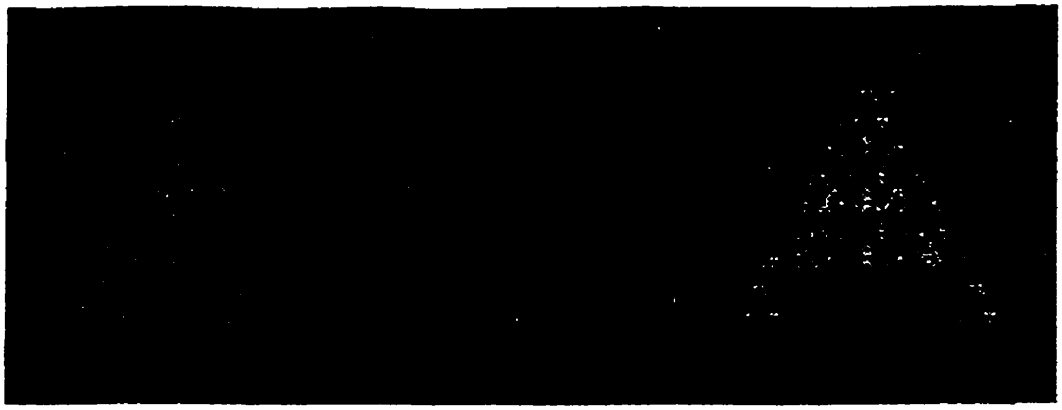


Image #1 Modulus

Image #2 Modulus

Image #3

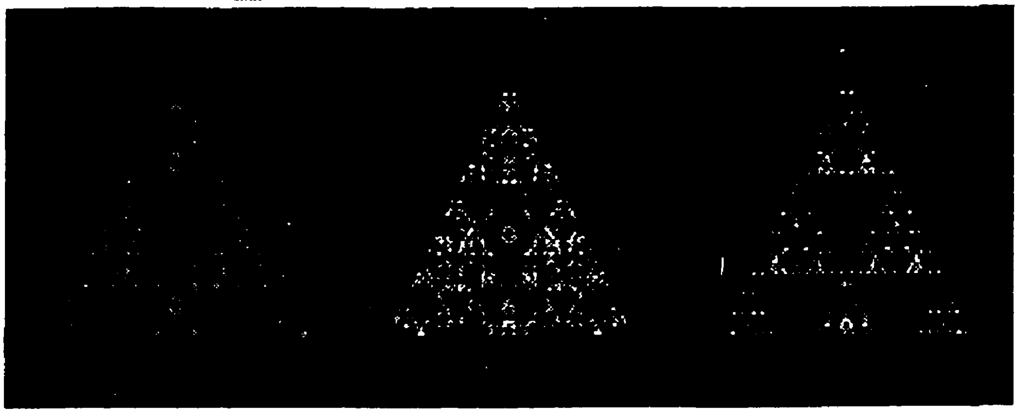


Image #4 Modulus

Image #5

Image #6

Reset

Pascal's Triangle | More Pascal's Triangle | Chinese Triangle | Blaise Pascal | Fibonacci | Probability

Please E-mail comments to: mmoor@sfu.ca

Answers for Coloring Pascal's Triangle

Image 1 was colored using modulus 4.

Image 2 was colored using modulus 2.

Image 3 was colored using modulus 7.

Image 4 was colored using modulus 3.

Image 5 was colored using modulus 9.

Image 6 was colored using modulus 5.

The different colors are formed by coloring numbers that have the same remainder when dividing the same color. For example in the modulus 4 drawing, numbers with a remainder of 0 are colored black, numbers with a remainder of 1 are colored red, numbers with a remainder of two are colored green and numbers with a remainder of three are colored blue. How many colors would you need to color the triangle if you used modulus 16?

More Pascal's Triangle

Related Topics

Now that you have learnt how to make Pascal's triangle and how to find patterns in the triangle by coloring, you may wish to explore other topics related to Pascal's triangle. The following pages contain information and activities that you can do that are related to Pascal and his triangle.

Blaise Pascal

This page is about Blaise Pascal; his childhood, mathematics and scientific acheivements, and other interests in his life.

Chinese Triangle

As previously mentioned, Blaise Pascal was not the first to discover the triangle that now bears his name. This page shows the triangle as the Chinese drew it and some history about the Chinese triangle.

Fibonacci Series

The Fibonacci series is a famous series of numbers that can be uesd in studying probability, fractals, the golden ratio and patterns in nature. This series of numbers can also be found in Pascal's triangle.

Probability

Pascal's triangle was made famous because he developed new ways in which to use the triangle. One of the areas that Pascal explored was using the triangle to solve probability problems. This page shows some ways in which you can use the triangle to solve probability problems.

Pascal's triangle | Coloring Pascal's triangle | Blaise Pascal | Chinese triangle | Fibonacci | Probability

Please E-mail comments to:
mmoor@sfu.ca

Blaise Pascal

(1623 - 1662)

<u>*"Man is but a reed, the weakest in nature, but he is a thinking reed"*</u>

Early Years

Blaise Pascal was born in Clermont-Ferrand, France in 1623. His mother died when he was 3 years old, leaving himself and two sisters to be raised by his dad Etienne Pascal. His father believed that in order to ensure Blaise not be overworked , his son should be schooled at home studying the languages only. His father was well versed in mathematics and considered the study of mathematics too stimulating for a young boy.

Studies in Geometry

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

At the age of 12, Blaise spurred on by the restrictions placed on his schooling, started to investigate the field of geometry. He gave up his play time for this new pursuit and in a few weeks had that sum of angles of a triangle is equal to 2 right angles. His father, recognizing the ability displayed by Blaise gave him a copy of Euclid's Elements. At the age of 14 Blaise began to attend the weekly meeting of geometricians and at the age of 16 he wrote an essay on Conic sections which included Pascal's theorem stating that "opposite sides of a hexagon inscribed in a conic, intersect in 3 collinear points". Pascal first used his now famous arithmetical triangle in 1653, but he did not give an account of his method until 1665.

Studies in Probability

Pascal's interest in probability was aroused by a problem posed by a gambler to him in 1654. Pascal relayed this problem to Pierre de Fermat, who also proceeded to work on the problem. Both Fermat and Pascal came up with solutions to the question, but the proofs for each was different. Pascal's arithmetical triangle can be used to illustrate solutions to many probability questions.

Studies in Physics

In 1646, Pascal began his barometric experiments which he continued for a further 8 years. His experiments on the statics of gases and liquids which he claimed contradicted the doctrine of horror vacui. In 1654, Pascal completed his Traite de l'equilbre which discussed the laws of hydrostatics and a description of the effects of the weight of air.

Calculating Machine

In 1645, after 3 years of work and 50 models, Pascal completed workon his calculating machine which could perform addition and ubtraction. This machine was a series of toothed wheels with each wheel having 10 teeth numbered 0 to 9. The wheels were arranged in a row with the first wheel represented the ones digits, the second the tens digits, the third the hundreds etc. To add two numbers such as 456 + 111, you would start by displaying 4, 5, and 6 in the appropriate viewing window and then moving each wheel by one tooth (+111).

Religious Background

Pascal became deeply religious in later life after coming in contact with the Jansenists. Jansenism was a movement in catholic theory in the 17th. and 18th. century and followers believed that they should serve God with all strength for gratitude. On November 23, 1654, Pascal believed he experienced a divine revelation when horses pulling his buggy bolted and fell over the parapet of the bridge at Neuilly. Pascal was saved because the trace connecting the buggy and horses broke and after this Pascal devoted the remaining 8 years of his life to spreading the word of God. He briefly revisted athematics again one night when suffering from a tooth-ache, again perceiving this to be a sign from God.

Further Investigations

The following links are to web pages with more detailed information on
Blaise Pascal and his discoveries:

1. Blaise Pascal; From "A Short Account of the History of Mathematics" by W.W. Rouse Ball.

http://www.maths.tcd.ie/pub/HistMath/People/Pascal/RouseBall/RB_Pascal.ht ml

2. Pascal, Blaise; Rice Universities site on Pascal.

http://es.rice.edu/ES/humsoc/Galileo/Catalog/Files/pascal_bla.html

3. The History of Computers:Blaise Pascal; This site discusses Blaise Pascal and his contribution to the development of computers.
http://www-stall.rz.fht-esslingen.de/studentisches/Computer_Geschichte/grp1/seite4.html

Pascal's triangle | Coloring Pascal's triangle | More Pascal's Triangle | Chinese triangle | Fibonacci | Probability

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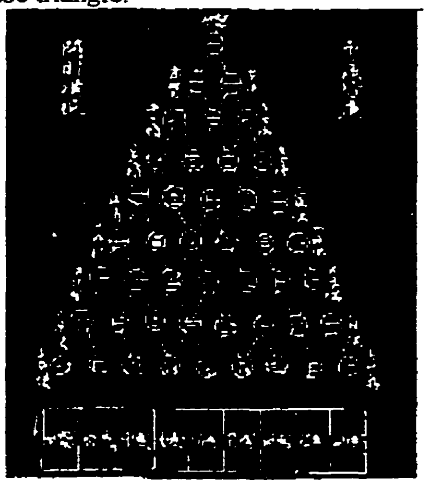
Chinese Triangle

History

Pascal was not the first to discover the arithmetical triangle. It first appeared in a book printed around 1303, three hundred years and two years befoere Blaise Pascal was born. However, Pascal extended the ideas presented by the Chinese triangle which is why it bears his name today. The following is a picture of the original Chinese triangle:



Investigations

1.Research or use the above diagram of the Chinese triangle to try to write the Chinese symbols for the following numbers:

-8
-11
-22
-35

2. Research other contributions by Chinese to the study of mathematics. Some good Web sites on the history of mathematics are found at:

History of Mathematics: by Region
http://aleph0.clarku.edu/~djoyce/mathhist/earth.html

Development of Mathematics in Ancient China
http://saxakali.com/COLOR_ASP/developcm3.htm

Another excellant site to explore is the following one on the Chinese abacus. The site allows you to interactively use an abacus. It also includes other fun activities such as how to build your own abacus using lego!

The Abacus
http://www.ee.ryerson.ca:8080/~elf/abacus/

Return to More Pascal's triangle

Pascal's triangle | Coloring Pascal's triangle | More Pascal's Triangle | Blaise Pascal | Fibonacci | Probability

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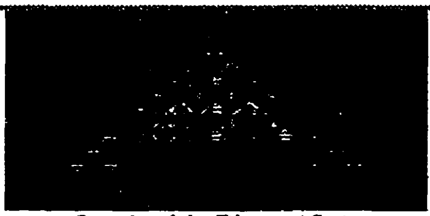

Pascal and the Fibonacci Series

History

Fibonacci was born in Pisa, Italy in 1175 AD. His real name was Leonardo da Pisa and he was the son of an Italian customs official and as a result traveled to many places in Africa, the Far East and the Middle East. In his travels, Fibonacci was introduced to the Hindu-Arabic number system. He wrote a book titled Liber abaci in which he outlined the Hindu-Arabic number system and how basic operations, algebra and geometry was performed with this number system. Europe was still using the Roman numeral system and it was as a result of Fibonacci's book and trade with Arabic countries that the switch was made to the Hindu-Arbic number system. Today, Fibonacci is most famous for the number sequence which resulted from the solution to a problem that was outlined in his book Liber abaci. The problem was: . 1. Suppose a one month old pair of rabbits (male and female) are too young to reproduce, but are mature enough to reproduce when they are two months old. Also assume that every month, starting from the second month, they produce a new pair of rabbits (male and female). 2. If each pair of rabbits reproduces in the same way as the above, how many pairs of rabbits will there be at the beginning of each month? 3. The solution to this problem gives the following sequence of numbers:

first month: 1 pair

second month: 1 pair

third month: 2 pairs

fourth month: 3 pairs

fifth month: 5 pairs

sixth month: 8 pairs

seventh month: 13 pairs
eight month : 21 pairs

The Fibonacci sequence, (1,1,2,3,5,8,13,21,34,55...), generated by this problem can also be found by taking the sum of the two proceeding terms to find the next term in the sequence.

Fibonacci & Pascal's Triangle

The Fibonacci sequence can be found in Pascal's triangle by looking at the sum of numbers in diagonal rows as shown in the following:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 7 1

The first diagonal row sums to 1, the second row to 1, the third row to 2, the fourth row to 3, the fifth row to 5, the sixth row to 8, the seventh row to 13, the eight row to 21, and the ninth row to 34 etc.

Fibonacci & the Golden Ratio

The Fibonacci sequence can also be used to find the Golden Ratio. The Golden Ratio is the ratio AC/AB when AB is the distance found on a line segment AC that makes the ratio AC/AB equal the ratio AB/BC.

A B C

If we look at the ratio of the Fibonacci sequence plus one to the Fibonacci sequence, we obtain the Golden Ratio when the Fibonacci sequence approaches infinity.

eg. $F_{n+1}/F_n = 1/1\ \ 2/1\ \ 3/1\ \ 5/3\ \ 8/5\ \ 13/8 \ldots$

$= 1, 2, 1.5, 1.6, 1.625, 1.6152\ldots$

Fibonacci & Nature

The Fibonacci sequence can be found naturrally occuring in many features of nature. If you count the number of petal-lie parts in a variety of flowers you find the Fibonacci sequence as illustrated below:

Lilies and Irises 3

Columbines, buttercups and larkspur 5

Delphiniums 8

Marigolds 13

Asters 21

Daisies 34,55,84

Fibonaacci numbers also appear in the arrangement of leaves around the stems of plants. If you start counting the leaves around the stem of a plant and assign the first leaf the number 0 and continue counting until you reach a leaf in line with the first you will likely get a Fibonacci number. For example on an Elm tree the number is 3, on a Cherry tree the number is 5 and on a Pear tree the number is 8. Spirals in objects such as Pine cones and Sunflowers seedheads also display a Fibonacci sequence. Spirals can be counted by looking at those that spiral to the right and those that spiral to the left. The number of spirals going in each direction tend to be Fibonaaci numbers for example a Sunflower seedhead may have 8 spirals curling to the right and 13 spirals curling to the left.

Fibonacci Investigations

Pascal's triangle | Coloring Pascal's triangle | More Pascal's triangle | Blaise Pascal | Chinese triangle | Probability

Please E-mail comments to:
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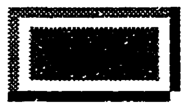
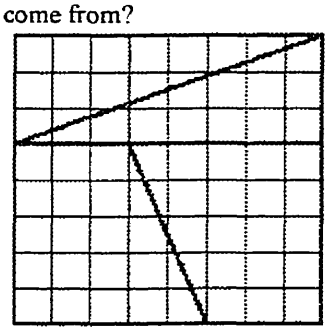
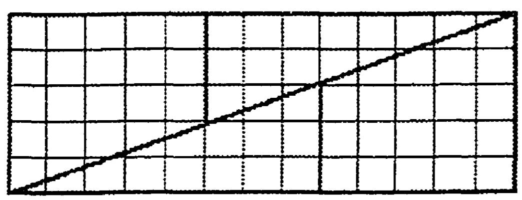
Fibonacci Investigations

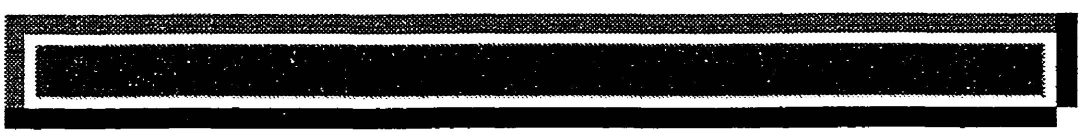
1. Fibonacci Magic: Start with any two numbers and generate a Fibonacci like sequence by adding the previous two terms to get the next term. Add up the first ten numbers in your sequence. Take the seventh term in the sequence and multiply it by eleven. What do you notice? Will this work for all Fibonacci-like sequence that you generate Can you prove why this works?



2. The Fibonacci Jigsaw Puzzle: Form a square by using the sum of any two consecutive Fibonacci numbers for each side length (in the blue diagram below the numbers 3 and 5 were used to make side lengths = 8). Draw two rectangles inside the square, one rectangle being the smaller Fibonacci number by the side length of the square and the other rectangle being the larger Fibonacci number by the side length of the square (see below). In the smaller rectangle draw in the diagonal of the rectangle from the bottom left to the top right. In the larger rectangle draw a line connecting the smaller Fibonacci number along one side of the smaller rectangle to the smaller Fibonacci number on the opposite side of the square (see below). Cut up the pieces formed and make them into the shape of a rectangle (as in red below). If you count the total area of the blue square and the total area of the red rectangle you will find you have added one square unit. Where did the extra square come from?

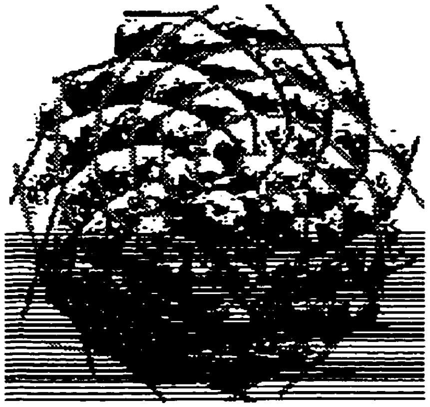






3. Fibonacci in nature: The Fibonacci sequence is found in many natural forms. Many seed heads often display the Fibonacci sequence in the number of spirals that go in each direction. The following diagram of the base of a pinecone is a good example. If you count the number of red

spirals (spirals going in one direction) and the number of green spirals (spirals going in the opposite direction) you will find that they are both Fibonacci numbers. Search for cones and other seed heads in your neighborhood and see if the number of spirals going in each direction are always Fibonacci numbers.



4. The following is a great link to a site on Fibonacci and his sequence of numbers. This site has many activities to try and further links to information on the Fibonacci sequence.

- The Fibonacci Numbers: This is a great site with many activities to try and links to more Fibonacci sites.
http:// www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html

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Fibonacci Magic: Activity #1

You should find that when you add up the first 10 integers in a Fibonacci like sequence the sum is always equal to the product of eleven times the seventh integer in the sequence. The following is one possible proof of this:

If the two starting numbers are "a" and "b" then the first ten integers in the sequence are:

a b a+b a+2b 2a+3b 3a+5b 5a+8b 8a+13b 13a+21b 21a+34

and the sum of this is:

55a+88b.

The product of eleven times the seventh integer is:

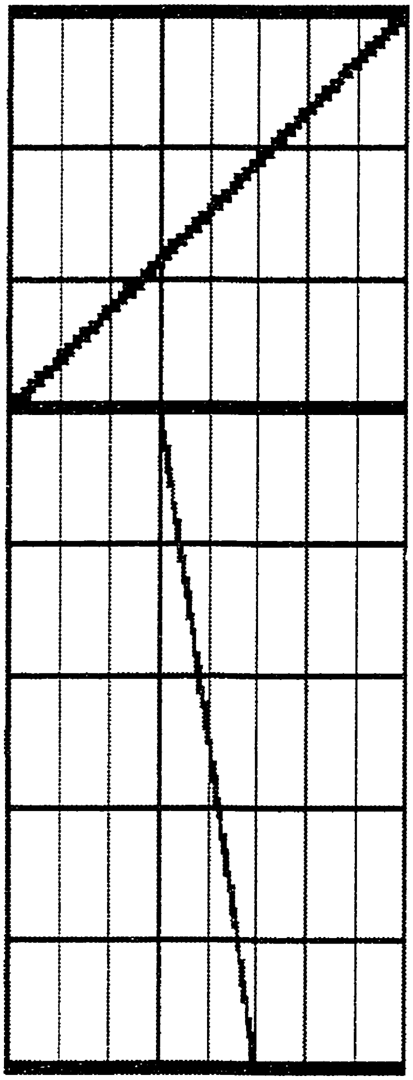
11(5a+8b)

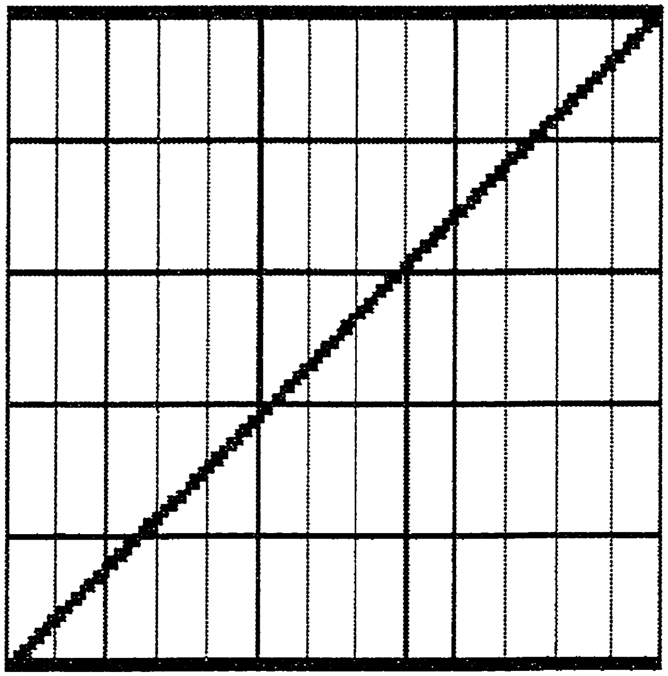
And this equals:

55a+88b.

Fibonacci Investigations:Activity #2

In order to try this activity you need to cut out the shapes in the first (blue) diagram and re-arrange them to form the rectangle in the second (red) diagram. To print this page choose "print" under the "file" menu.





Pascal's Triangle and Probability

Using the Triangle to find Probablilities

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Pascal developed his triangle in the course of his work investigating probability and it should be no surprise that it is well suited to solving problems involving probability.

The triangle can be used to determine questions involving possible combinations of objects. For example if there were four coins tossed into the air the posible combinations of heads and tails would be:

4 heads : 1

3 heads, 1 tail : 4

2 heads, 2 tails : 6

3 heads, 1 tail : 4

4 tails : 1

The possible combinations can be found in the 5th. row of Pascal's Triangle.

Problems to Solve

1.If 5 coins were tossed in the air what would be the possible combinations of heads and tails? Can you find this sequence in Pascal's triangle? Does this probability work in practise? Try it by flipping 5 coins 100 times. Record the number of heads and tails each time. How close do you come to your predictions?

2. An organizer of a doubles tennis tournament wants to randomly match 10 people in pairs. The organizer intends to do this by placing all ten names into a hat and drawing out names two at a time. What are the total possible combinations for the:

- first draw?
- second draw?
- third draw?
- fourth draw?
- fifth draw?

Can you use Pascal's triangle to help find the answers to this problem? Click on the "Check" button for answers to these questions.

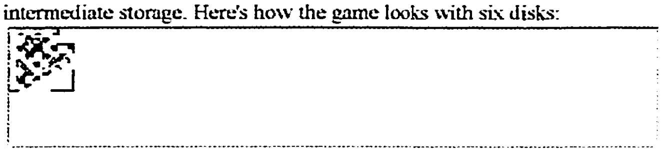
Check

The Tower of Hanoi Game

The Legend. In an ancient city in India, so the legend goes, monks in a temple have to move a pile of 64 sacred disks from one location to another. The disks are fragile; only one can be carried at a time. A disk may not be placed on top of a smaller, less valuable disk. And, there is only one other location in the temple (besides the original and destination locations) sacred enough that a pile of disks can be placed there.

So, the monks start moving disks back and forth, between the original pile, the pile at the new location, and the intermediate location, always keeping the piles in order (largest on the bottom, smallest on the top). The legend is that, before the monks make the final move to complete the new pile in the new location, the temple will turn to dust and the world will end. Is there any truth to this legend?

The Game. There's a game based on this legend. You have a small collection of disks and three piles into which you can put them (in the physical version of this game, you have three posts onto which you can put the disks, which have holes in the centre). The disks all start on the leftmost pile, and you want to move them to the rightmost pile, never putting a disk on top of a smaller one. The middle pile for intermediate storage. Here's how the game looks with six disks:



The object of the game is to transfer the disks to the new location in the least number of turns. You may play the game interactively on the computer by clicking below or cut out your own disks to play with.

Click here for interactive game

Once again, the answer for the least number of moves can be found in Pascal's triangle. As you play the game keep track of the least number of moves for each starting number of disks and see if you can find this answer in the triangle. Click below to see how to use the triangle to find the answers.

Check

Return to More Pascal'sTriangle

Pascal's triangle | Coloring Pascal's triangle | Blaise Pascal | Chinese triangle | Fibonacci | Probability

1. Flipping Coins: To find the number of possible combinations and the likelihood of flipping each combination look in the sixth row of Pascal's triangle. The answers are:

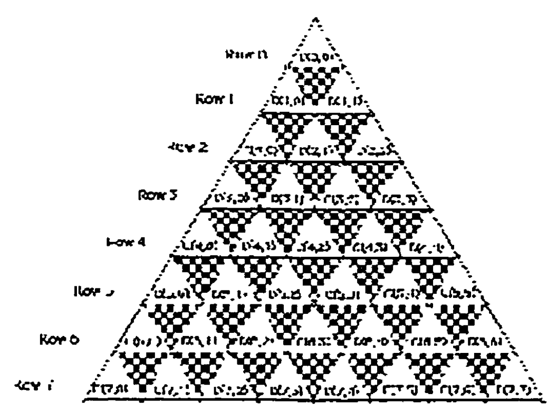
- 5 heads: 1
- 4 heads, 1 tail: 5
- 3 heads, 2 tails: 10
- 2 heads, 3 tails: 10
- 1 head, 4 tails: 5
- 5 tails 1

To find the probability of flipping each combination divide the likelihood of flipping each combination by the total possible outcomes. Therefore the probability of flipping 5 heads would be 1/32.

2. The Tennis Tournament: Again, we can find the answers for the tennis tournament draw in Pascal's triangle. Because we remove two names from the hat each time we draw, the possible combinations are always changing. On the first draw we are choosing 2 names out of a total of ten. We would call this 10 choose 2 and the answer to this is in the tenth row of Pascal's triangle **if we call the first row of the triangle row zero.** On the second draw we are choosing 2 names out of a total of eight (8 choose 2) and the answer for this is in the eighth row of Pascal's triangle. On the third draw we are choosing 2 names out of a total of six (6 choose 2) and the answer is found in the sixth row of Pascal's triangle. On the fourth draw we are choosing 2 names out of a total of four (4 choose 2) and the answer is found in the fourth row of Pascal's triangle. Lastly, we are choosing two names out of a total of two and the answer is found in the second row of Pascal's triangle. Here are the answers for the number of combinations possible in each draw:

- first draw 45 combinations
- second draw 28 combinations
- third draw 15 combinations
- fourth draw 6 combinations
- fifth draw 1 combination

The following diagram shows the location in Pascal's triangle for each of the answers. (C(6,2) represents 6 choose 2 and C(4,2) represents 4 choose 2)



3. The Tower of Hanoi: The following table shows the minimum number of moves for games starting with 1 to 10 disks.

Starting # of disks	Minimum # of moves
1	1
2	3
3	7
4	15
5	31
6	63
7	127
8	255
9	511
10	1023

To find these answers these answers in Pascal's triangle you need to number the rows in the triangle starting with the top row as zero and continuing downwards. Then to find the minimum numbers of moves, start with the row that matches your starting number of disks. Then add all the numbers in that row and subtract 1 from the sum. The following is an example for a game starting with 6 disks:

Row 0	1
Row 1	1 1
Row 2	1 2 1
Row 3	1 3 3 1
Row 4	1 4 6 4 1
Row 5	1 5 10 10 1
Row 6	1 6 15 20 15 6 1

Adding the numbers in the sixth row we get 64 and subtract 1 equals 63.

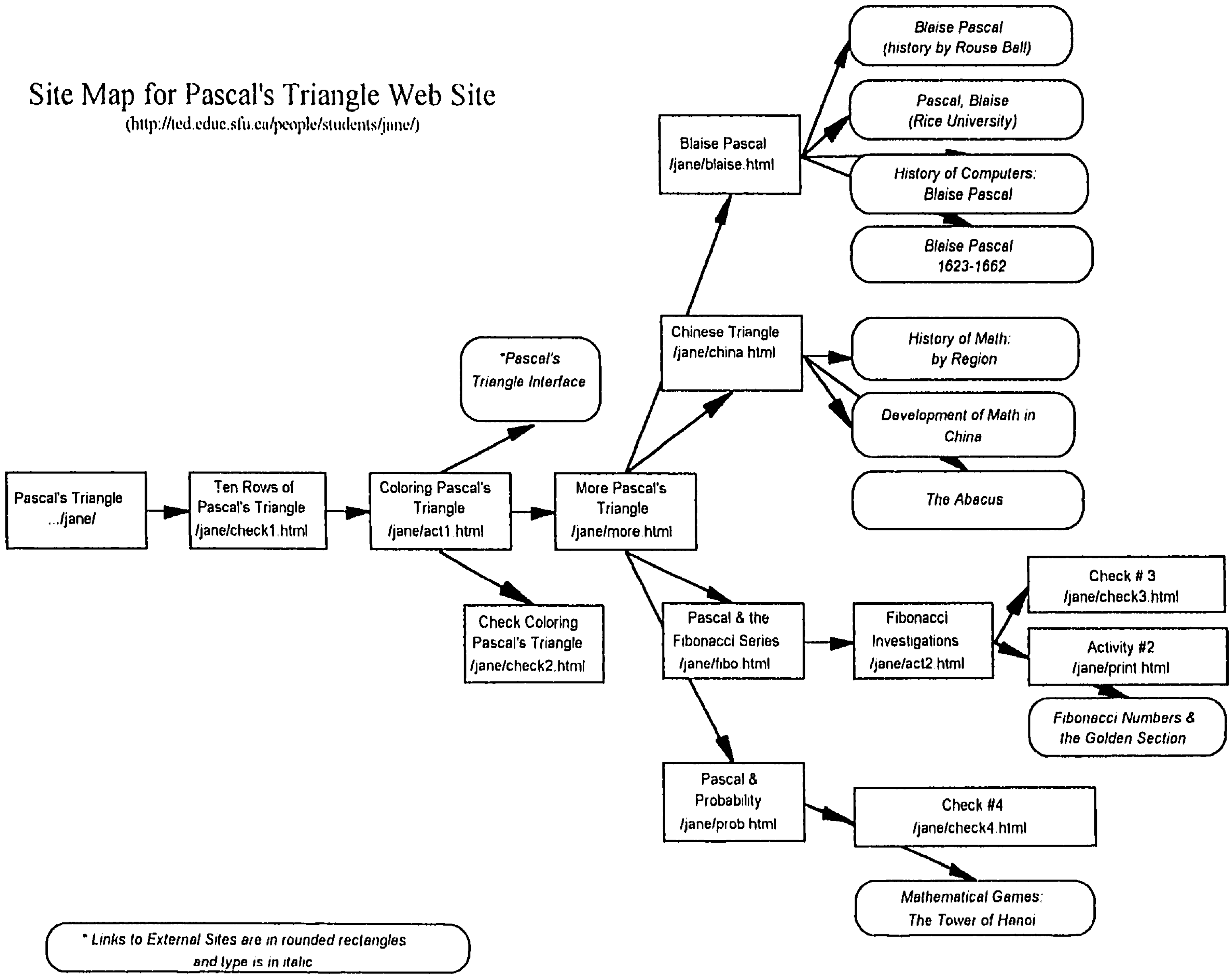
Back to Probability and Pascal | Return to More Pascal's Triangle

APPENDIX B

A Site Map for Pascal's Triangle

Site Map for Pascal's Triangle Web Site

(http://ted.educ.sfu.ca/people/students/jane/)



APPENDIX C

Assignment Instructions Given to MATH 190 Students

Internet Exploration

Welcome to the World Wide Web. This project will give you an opportunity to explore a Web site that has been designed for use by grade 7,8, and 9 math students. For those of you that have had experience in exporing the World Wide Web, you will have found that most educational sites are either "virtual offices, where instructors display materials such as course outlines and assignments, or "virtual libraries", where a variety of information (mainly text based) is displayed for research purpores. This site is innovative in that it attempts to become a "virtual classroom" that allows students to explore and investigate a mathematical concept. The site is based on the theme of Pascal's triangle but because of the interactive nature of the Web you will find many paths that you can follow that will lead you in a variety of different directions. The U.R.L. for the site is: "http://ted.educ.sfu.ca/people/students/jane/".

Your task for completing this project is made up of three components:

- a journal detailing where you have been and what you have done while using the site
- your solution to one of three mathematical problems that are found on the site
- a summary of your reflections on the site

The Journal: The purpose of the journal is to keep a record of **all** pages and external sites that you visit as well as the activities that you engage in and reflections you make as you use the site. Marking of the journal will consider the thoroughness and organization of the journal and the detail and clarity of the reflections. Below is a sample exerpt from a double-entry journal whose format you may follow is you wish:

Coloring Pascal's Triangle U.R.L.: "http://ted.educ.sfu.ca/ people/students/jane/ act1.html"	This page discusses using "modulus" to color and find patterns in Pascal's triangle. I used the interactive site to experiment with coloring the triangle. I found the modulus concept difficult to grasp at first but once I spent some time exploring ...

The Mathematical Problem: The purpose of this part of the project is to explore one of the problems presented in the site in more detail. You may choose one of the three problems to explore:

1. <u>Using modulus to color and find patterns in Pascal's triangle.</u> This problem is found on the "Coloring Pascal's triangle" page at "http://ted.educ.sfu.ca/people/students/jane/act1.html". You should discuss what the term "modulus" means, how it is used to color the triangle, and

similarities and differences found in the patterns that are produced.

2. The Tower of Hanoi Problem. This problem is found on the "Probability and Pascal" page found at http://ted.educ.sfu.ca/people/students/jane/prob.html". Use the interactive site to play the game and then discuss the solutions for the minimum number of moves that can be made for games that start with 1 to 10 disks. Explain how Pascal's triangle can be used to find the solutions for the game.

3. Fibonacci and the Jigsaw Puzzle. This problem can be found on the "Fibonacci Investigations" page at "http://ted.educ.sfu.ca/people/students/jane/act2.html". Print out the page that is provided and try the puzzle. Discuss whether this puzzle will work with other numbers in the Fibonacci series and why the areas of the starting square and resulting rectangle are not the same.

Your discussion of the problems should be limited to one or two pages including any illustrations that you may wish to make. Marking will consider demonstration of your mathematical understanding and the clarity with which this is portrayed.

3. The Summary of your Reflections. The summary should focus on your experiences learning mathematics through the "virtual classroom" of the World Wide Web. Some sample questions that could be addressed are:

-What benefits/drawbacks are there to using a "virtual classroom" over the more traditional classroom?
-Can all mathematics be learnt through this medium or is its role in education limited?
-How much teacher support is needed for students learning mathematics through the World Wide Web?
-Does the interactive nature of the Web help or hinder the learning of mathematics?
-What is the level of difficulty of the mathematics that is presented on this site?
-Does the thematic nature of the site make it difficult to use in classrooms structured around specific curricula?

Marking of the summary will consider the relevance, thoroughness and clarity of your comments. The length of the summary should be restricted to a single page.

I hope you enjoy using the site and any questions or comments that you have can be e-mailed to me by following the link at the bottom of each page.

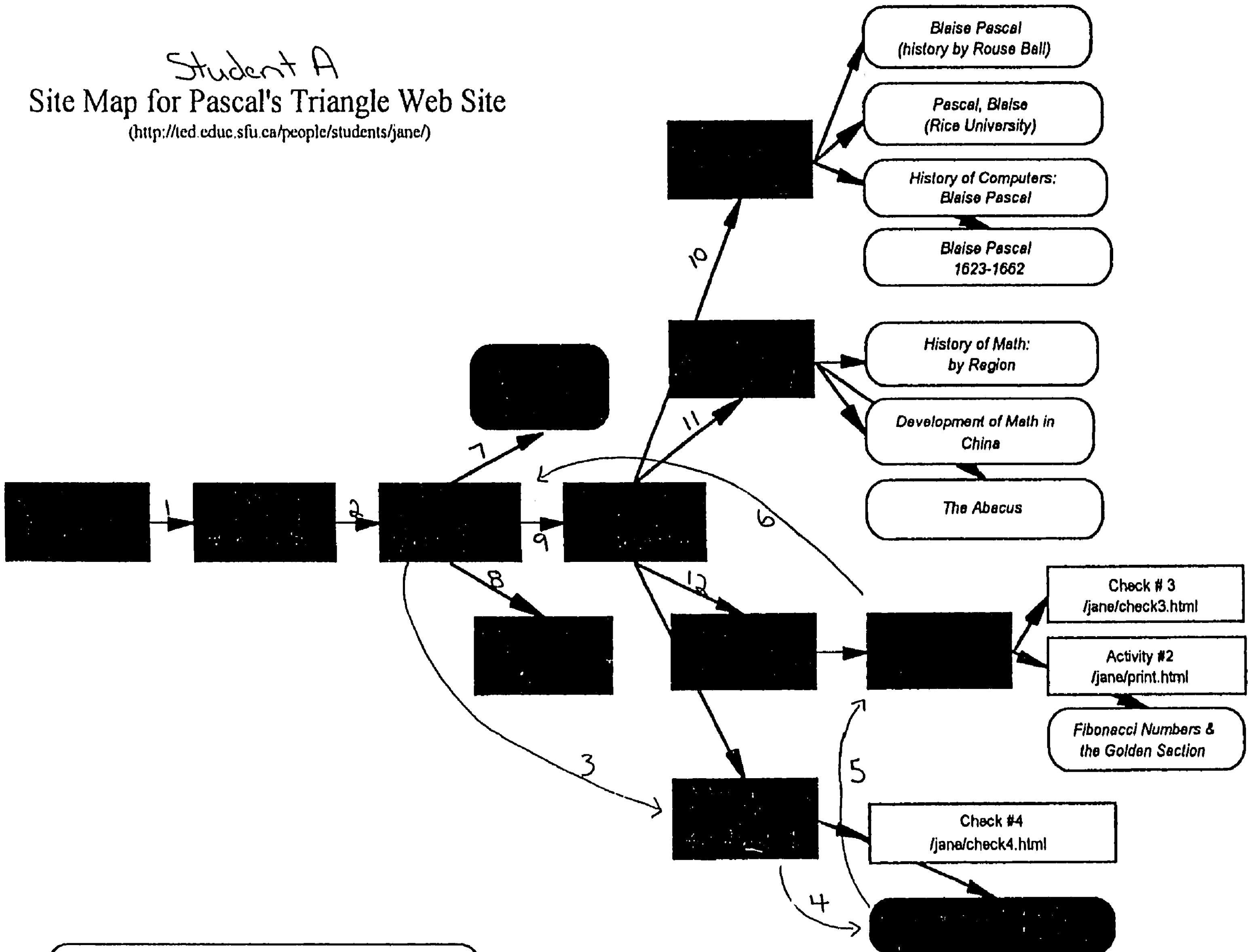
APPENDIX D

Representative Navigational Maps

Student A

Site Map for Pascal's Triangle Web Site

(http://ted.educ.sfu.ca/people/students/jane/)

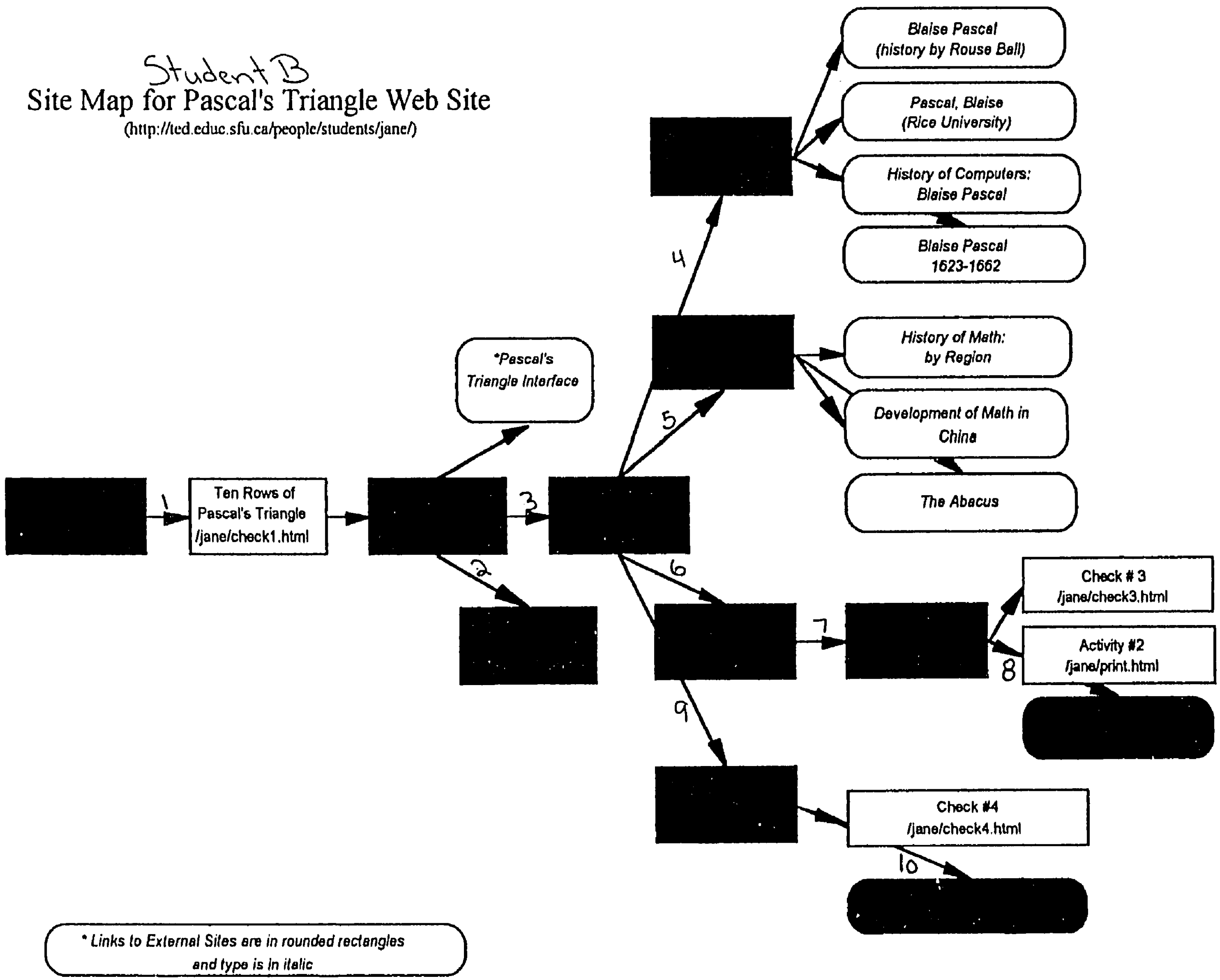


** Links to External Sites are in rounded rectangles and type is in italic*

Student B

Site Map for Pascal's Triangle Web Site

(http://ted.educ.sfu.ca/people/students/jane/)



Student C

Site Map for Pascal's Triangle Web Site

(http://ted.educ.sfu.ca/people/students/jane/)

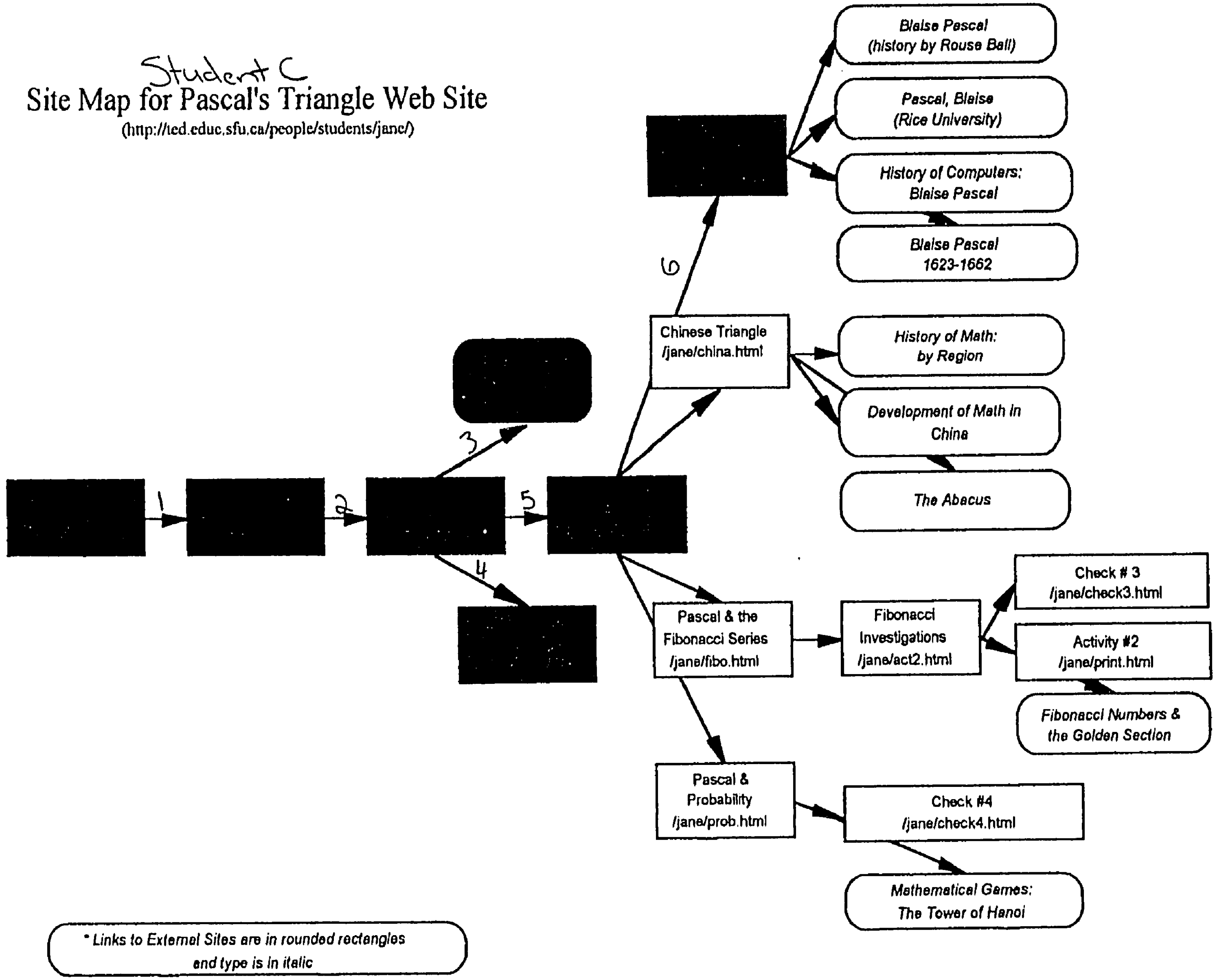


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IMAGE EVALUATION TEST TARGET (QA-3)

150mm

6"

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